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CONSTRUCTION AND SELECTION OF CHAIN SAMPLING PLAN WITH ZERO – INFLATED POISSON DISTRIBUTION

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Abstract:

In this article given a construction procedure of attribute Zero – Inflated Poisson chain sampling plan-1 for variable fraction defective is presented using stochastic differential equations. An iterative procedure for finding the parameters of the plan satisfying the given conditions with respect to producer quality level is given. Tables are constructed for easy selection of the parameters which are readily available to apply in the shaft floors of production process. The performance of the chain sampling plan for variable fraction defective is also discussed by determining the operating characteristic function. The procedure is developed to draw an Average Outgoing Quality Level (AOQL), curve by using Zero – Inflated Poisson chain sampling plan-1 and compare the different procedures in order to show the ambiguity of the procedure and their results.

Key Words: Chain Sampling Plan, Zero – Inflated Poisson Distribution & Average Outgoing Quality Level (AOQL).

Introduction:

The Modern developments in technology help to improve the quality of products and their production. Improvement in product quality is also due to the well monitoring of the production process. Products not meeting the specified quality standards in these situations is a rare phenomenon. Sampling inspection carried out in these case may result with the information consisting many zeros which are the values of the number of nonconforming product. The Zero-Inflated Poisson (ZIP) distribution can be used as the appropriate probability distribution to data consisting many over dispersed zeros. ZIP distribution has been used in a wide range of disciplines such as agriculture, epidemiology, econometrics, public health, process control, medicine, manufacturing, etc. Some of the applications of ZIP distribution can be found in Bohning et al. (1999), Dodge (1955) has proposed Chain Sampling Plan in which allows significant reduction in sample size and the condition for a continuing succession of lots from a stable and trusted supplier. Lambert (1992), and Yang et al. (2011). Construction of control charts using ZIP distribution are discussed in Sim and Lim (2008).Some theoretical aspects of ZIP distributions are mentioned in McLachlan and Peel (2000). Single sampling plans by attributes under the conditions of Zero – inflated Poisson distribution are determined by Loganathan and Shalini (2013), Suresh and Latha (2002) have given a procedure and tables for the selection of Bayesian chain sampling plan-1. Soundararajan (1978 a, b) has described procedures and tables for construction and Selection of Chain sampling plans (ChSP-1) indexed by specified parameters.

Conditions for Application of ChSP -1:

The cost of destructiveness of testing is such that a relatively small sample sizes is necessary, although other factors make a large sample desirable.

- ✓ The product to be inspected comprises a series of successive lots produced by a continuing process.
- ✓ Normally lots are expected t be of essentially the same quality.
- ✓ The consumer has faith in the integrity of the producer.

Operating Procedure:

The plan is implemented in the following way:

- ✓ For each lot, select a sample of n units and test each unit for conformance to the specified requirements.
- ✓ Accept the lot if d (the observed number of defectives) is zero in the sample of n units, and reject if d>1.
- ✓ Accept the lot if d is equal to 1 and if no defectives are found in the immediately preceding i samples of size n.

Dodge (1955) has given the operating characteristic function of ChSP-1 as $P_a(p) = P_0 + P_1(P_0)^i$
Where Pj = probability of finding j nonconforming units in a sample of n units for j = 0.1.
The Chain sampling Plan is characterized by the parameters n and i. When i=0 , the OC function of a ChSP -1 plan reduces to the OC function of the Single Sampling Plan with acceptance number zero and when i = 0, the OC function of ChSP-1 plan reduces to the OC function of the Single Sampling Plan with acceptance number 1.

Operating Characteristic Function of ZIP Model:

The OC function is defined as

$$P_a(p) = P[X \leq c] \qquad (1)$$

Where p is the probability of fraction defective

The numbers of defects are zero for many samples there may consider Zero – inflated Poisson probability distribution. The probability mass function of the ZIP (φ, λ) distribution is given by Lambert (1992) and McLachlam and peel (2000)

$$P(X = x \mid \phi, \lambda) = \varphi \, f(x) + (1-\varphi) P(X=x \mid \lambda) \qquad (2)$$

Where

$$f(x) = \begin{cases} 1, & if\ x = 0 \\ 0, & if\ x \neq 0 \end{cases} \quad \text{and}$$

$$P(X = x / \lambda) = \frac{e^{-\lambda}\lambda^x}{x!}, \qquad \text{when } x = 0, 1, 2, \ldots$$

The probability mass function can also be expresses as

$$P(X = x \mid \varphi, \lambda) = \begin{cases} \varphi + (1-\varphi)e^{-\lambda} & when \quad x = 0 \\ (1-\varphi)\dfrac{e^{-\lambda}\lambda^x}{x!}, & when \quad x = 1, 2, \ldots, 0 < \varphi < 1,\ \lambda > 0 \end{cases}$$

In this distribution, φ may be termed as the mixing proportion. φ and λ are the parameters of the ZIP distribution. According to McLachlan and Peel (2000), a Zip distribution is a special kind of mixture distribution.

The OC function of the conditions of ZIP (φ, λ) distribution can be defined as

$$P_a(p) = \sum_{x=0}^{c} P(X = x \mid \phi, \lambda)$$

$$P_a(p) = \varphi + (1-\varphi)e^{-\lambda} + \sum_{x=1}^{c}(1-\varphi)\frac{e^{-\lambda}\lambda^x}{x!}, \quad x > 0,\ \lambda > 0,\ 0 < \varphi < 1. \qquad (3)$$

Where $\lambda = np$

Chain Sampling Plans (ChSP-1) with Zero- Inflated Poisson Distribution:

The probability of acceptance for chain sampling plan of type ChSP- 1 based on Zero- inflated Poisson distribution

$$P_a(p) = (\varphi + (1-\varphi)e^{-np}) + (\varphi + (1-\varphi)e^{-np})^{i+1} + (1-\varphi)e^{-np}\, np\, (\varphi + (1-\varphi)e^{-np})^{i} \qquad (4)$$

Average Outgoing Quality Limit (AOQL):

The AOQL of a sampling plan is maximum value on the AOQ curve. It is applicable for defective units, defects per unit, and defects per quantity. It is expressed as either a defective rate (fraction defective, percent defective, dpm) or as a defect rate (defects per unit, defects per 100 units, dpm). The AOQ curve gives the average outgoing quality (left axis) as a function of the incoming quality (bottom axis). The AOQL is the maximum or worst possible defective or defect rate for the average outgoing quality. Regardless of the incoming quality, the defective or defect rate going to the customer should be no greater than the AOQL over an extended period of time. Individual lots might be worst than the AOQL but over the long run, the quality should not be worse than the AOQL.

Thus Chain sampling plans Zero- inflated Poisson distribution Average Outgoing Quality (AOQ) is approximated obtained by

$$\text{AOQ} = p P_a(p)$$

$$\text{nAOQ} = (np\varphi + np(1-\varphi)e^{-np}) + np(\varphi + (1-\varphi)e^{-np})^{i+1} + (1-\varphi)e^{-np}(np)^2(\varphi + (1-\varphi)e^{-np})^{i} \qquad (5)$$

Differentiating AOQ with respect to *np* and equating to 0, the value of Average Outgoing limit (AOQL) can be obtained by solving the equation.

$$\varphi + (1-\varphi)e^{-np}((\varphi + (1-\varphi)e^{-np})^{i} + np(1-\varphi)e^{-np}(1 - np - i)\ (\varphi + (1-\varphi)e^{-np})^{i-1}$$
$$-(np)^2 e^{-2np}(1-\varphi)^2\ i\ (\varphi + (1-\varphi)e^{-np} + 1)) - np(1-\varphi)e^{-np} = 0 \qquad (6)$$

From Equation (6) the values of *np (=npm)* can be calculated for different values of φ and i .Substituting *npm* in equation (5) nAOQL values are obtained.

Comparison with Poisson Chain Sampling Plan (ChSP – 1):

From Table 1 for fixed value of *φ*, *i*, the *np* value is must greater than Poisson plan for higher value of probability of acceptance. When the probability of acceptance is very low the *np* value for Zero – Inflated Poisson model is greater than but closer to that of Poisson model.

Regarding Average Outgoing Quality Limit (AOQL) the small value of *i*. The Average Outgoing Quality limit (AOQL) is Zero – Inflated Poisson model is should be higher than the Poisson model as φ increases the value of Average Outgoing Quality Limit (AOQL) is also increases, as *i* value increases the Average Outgoing Quality Limit (AOQL) for the Zero – Inflated Poisson model approaches to the Poisson model.

Table 1: Chain Sampling Plan under Zero - Inflated Poisson

φ		$P_a(p)$					
	i	**0.99**	**0.95**	**0.90**	**0.50**	**0.10**	**0.05**
0.0001	1	0.6343	0.6691	0.7147	1.2014	2.5486	3.1605
	2	0.4658	0.4937	0.5305	0.9494	2.3343	3.0074
	3	0.3774	0.4019	0.4344	0.8334	2.3067	2.9981
	4	0.3214	0.3438	0.3738	0.7711	2.3038	2.9976
	5	0.2822	0.3031	0.3315	0.7361	2.3035	2.9976
	6	0.2528	0.2727	0.3000	0.7163	2.3034	2.9976
	7	0.2299	0.2490	0.2755	0.7054	2.3034	2.9976
	8	0.2114	0.2299	0.2557	0.6995	2.3034	2.9976
	9	0.1962	0.2141	0.2395	0.6964	2.3034	2.9976
0.001	1	0.6350	0.6699	0.7155	1.2033	2.5583	3.1795
	2	0.4663	0.4943	0.5311	0.9508	2.3426	3.0248
	3	0.3778	0.4023	0.4349	0.8346	2.3149	3.0154
	4	0.3218	0.3441	0.3742	0.7722	2.3119	3.0149
	5	0.2825	0.3034	0.3319	0.7370	2.3116	3.0149
	6	0.2531	0.2730	0.3003	0.7172	2.3116	3.0149
	7	0.2302	0.2492	0.2757	0.7063	2.3116	3.0149
	8	0.2117	0.2301	0.2560	0.7004	2.3116	3.0149
	9	0.1964	0.2143	0.2397	0.6973	2.3116	3.0149
0.01	1	0.6424	0.6779	0.7242	1.2224	2.6612	3.3936
	2	0.4715	0.4998	0.5372	0.9646	2.4307	3.2197
	3	0.3819	0.4067	0.4397	0.8462	2.4013	3.2093
	4	0.3251	0.3478	0.3783	0.7827	2.3982	3.2088
	5	0.2854	0.3066	0.3354	0.7469	2.3979	3.2088
	6	0.2557	0.2758	0.3035	0.7268	2.3978	3.2088
	7	0.2325	0.2518	0.2786	0.7156	2.3978	3.2088
	8	0.2138	0.2324	0.2586	0.7097	2.3978	3.2088
	9	0.1983	0.2164	0.2422	0.7065	2.3978	3.2088
0.05	1	0.6775	0.7156	0.7656	1.3158	3.3414	3.2978
	2	0.4961	0.5262	0.5661	1.0316	2.9914	3.1290
	3	0.4011	0.4274	0.4626	0.9022	2.9493	3.0636
	4	0.3411	0.3651	0.3974	0.8331	2.9449	3.0366
	5	0.2992	0.3216	0.3521	0.7943	2.9444	3.0258
	6	0.2679	0.2891	0.3183	0.7725	2.9444	3.0258
	7	0.2435	0.2638	0.2921	0.7605	2.9444	3.0258
	8	0.2238	0.2434	0.2710	0.7541	2.9444	3.0258
	9	0.2076	0.2266	0.2537	0.7507	2.9444	3.0258
0.09	1	0.7169	0.7580	0.8121	1.4263	4.7573	4.3167
	2	0.5233	0.5556	0.5984	1.1095	4.6560	4.1290
	3	0.4224	0.4504	0.4879	0.9667	4.5253	4.0260
	4	0.3588	0.3843	0.4187	0.8910	4.5123	4.0235
	5	0.3144	0.3381	0.3705	0.8486	4.5110	4.0220
	6	0.2813	0.3038	0.3347	0.8248	4.5108	4.0220
	7	0.2555	0.2770	0.3070	0.8118	4.5108	4.0220
	8	0.2348	0.2555	0.2847	0.8047	4.5108	4.0220
	9	0.2177	0.2378	0.2664	0.8011	4.5108	4.0220

Table: 2 Certain Parametric values Chain Sampling Plan (ChSP-1) with ZIP Model

φ	i	np_1	np_2	np_m	nAOQL	p_2 / p_1	AOQL/ p_1
0.0001	1	0.6691	2.5486	0.8325	0.6508	3.8090	0.9726
	2	0.4937	2.3343	0.7070	0.4935	4.7282	0.9996

	3	0.4019	2.3067	0.7109	0.4201	5.7395	1.0453
	4	0.3438	2.3038	0.8187	0.3860	6.7010	1.1227
	5	0.3031	2.3035	0.9234	0.3739	7.5998	1.2336
	6	0.2727	2.3034	0.9696	0.3701	8.4466	1.3572
	7	0.2490	2.3034	0.9880	0.3688	9.2506	1.4811
	8	0.2299	2.3034	0.9954	0.3683	10.0191	1.6020
	9	0.2141	2.3034	0.9985	0.3682	10.7585	1.7198
0.001	1	0.6699	2.5583	0.8347	0.6519	3.8189	0.9731
	2	0.4943	2.3426	0.7097	0.4945	4.7392	1.0004
	3	0.4023	2.3149	0.7153	0.4212	5.7542	1.0470
	4	0.3441	2.3119	0.8259	0.3874	6.7187	1.1258
	5	0.3034	2.3116	0.9306	0.3738	7.6190	1.2320
	6	0.2730	2.3116	0.9761	0.3678	8.4674	1.3473
	7	0.2492	2.3116	0.9941	0.3652	9.2761	1.4655
	8	0.2301	2.3116	0.9963	0.3641	10.0461	1.5824
	9	0.2143	2.3116	1.0044	0.3636	10.7867	1.6967
0.01	1	0.6779	2.6612	0.8571	0.6624	3.9257	0.9771
	2	0.4998	2.4307	0.7382	0.5048	4.8633	1.0100
	3	0.4067	2.4013	0.7643	0.4328	5.9044	1.0642
	4	0.3478	2.3982	0.9047	0.4022	6.8953	1.1564
	5	0.3066	2.3979	0.9346	0.3927	7.8209	1.2808
	6	0.2758	2.3978	0.9788	0.3899	8.6940	1.4137
	7	0.2518	2.3978	0.9822	0.3891	9.5226	1.5453
	8	0.2324	2.3978	0.9983	0.3888	10.3176	1.6730
	9	0.2164	2.3978	1.0674	0.3887	11.0804	1.7962
0.05	1	0.7156	3.3414	0.9822	0.7156	4.6694	1.0000
	2	0.5262	2.9914	0.9291	0.5600	5.6849	1.0642
	3	0.4274	2.9493	0.9563	0.5041	6.9006	1.1795
	4	0.3651	2.9449	0.9623	0.4939	8.0660	1.3528
	5	0.3216	2.9444	0.9764	0.4922	9.1555	1.5305
	6	0.2891	2.9444	0.9783	0.4919	10.1847	1.7015
	7	0.2638	2.9444	0.9854	0.4918	11.1615	1.8643
	8	0.2434	2.9444	0.9930	0.4916	12.0970	2.0197
	9	0.2266	2.9444	1.0620	0.4912	12.9938	2.1677
0.09	1	0.7580	4.7573	0.9973	0.7877	6.2761	1.0392
	2	0.5556	4.6560	0.9224	0.6255	8.3801	1.1258
	3	0.4504	4.5253	0.9374	0.5667	10.0473	1.2582
	4	0.3843	4.5123	0.9418	0.5437	11.7416	1.4148
	5	0.3381	4.5110	0.9592	0.5412	13.3422	1.6007
	6	0.3038	4.5108	0.9750	0.5410	14.8479	1.7808
	7	0.2770	4.5108	0.9882	0.5408	16.2845	1.9523
	8	0.2555	4.5108	0.9989	0.5405	17.6548	2.1155
	9	0.2378	4.5108	1.0079	0.5400	18.9689	2.2708

Conclusion:

In a well – monitored process, most of the products will meet the specified quality standards. Occurrence of Zero- nonconforming per product would be more frequent in this sampling inspection. A zero inflated model is the appropriate probability distribution to the number of nonconformities per product manufactured in such production process. The Chain sampling plan gives more pressure on the producer if the quality deteriorates. These plans provide consumer an assurance regarding the outgoing quality or the quality of the lot after the inspection. Hence one can recommend this type of sampling plans for better quality control practice. This plan will be more useful to the quality control practioners to meet out the consumer requirements**.**

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