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TRIPLE CONNECTED DOMINATION NUMBER OF A GRAPH

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Abstract:

The concept of triple connected graphs with live application was introduced in by considering the available of roots containing any three vertices of a graph G. In this thesis, we introduce a new dominating parameter, called Smarandachely triple connected domination number of a graph. A subset S of V of a nontrivial G-graph is said to be Smarandachely triple connected dominated set, if S is a dominating set and the induced sub graph S is triple connected. The nominal cardinality take over all Smarandachely triple connected dominant sets is called the Smarandachely triple connected domination number and is denoted by γtc. We assumed this number for some standard graphs and obtain sustained bounds for general graphs. It's have the relationship with other graph theoretical parameters also investigated.

Key Words: Domination Number, Triple Connected Graph & Smarandachely Triple Connected Domination Number.

Introduction:

One of the youngest and dynamic growing areas in graph theory is the study of domination. It takes back to 1850's with the study of the problem of determining the minimum number of queen which are necessary to cover an n*n chessboard. More than 50 types of domination parameters have been studied by different authors. Ore, Berg introduced the concept of domination sets. Extensive research activity is going on in Domination set of graphs. Acharya B. D, Sampath Kumar. E, V. R Kulli, Waliker H. B are some of the Indian Mathematicians who have made substantial contribution to the study of domination in graphs. Domination is applied in many fields. Some of them are.

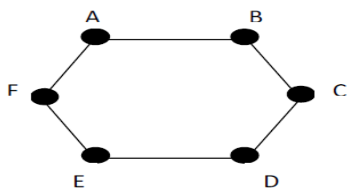
- ✓ Communication network
- ✓ Facility location problem
- ✓ Land surveying
- ✓ Routings etc.,

Basic Definitions:

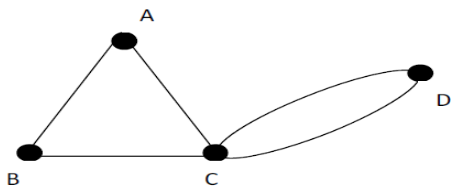
Graph: A graph consists of a set $V = \{v_1, v_2, \ldots, v_n\}$ called various certain vertices and another set $E = \{e_1, e_2, \ldots, e_m\}$ whose element are called edges such that each edge e_k is identified with an unordered pair (v_i, v_j) of vertices, the vertices (v_i, v_j) associated with of the edge $e_k$are called the end vertices of the edge e_k.

Order and Size of a Graph: The number of vertices in V(G) is called the order of G and the number of edges in E(G) is called the size of G.

Simple Graph: A graph which has no loops and multiple edges is called a basic situated simple graph.

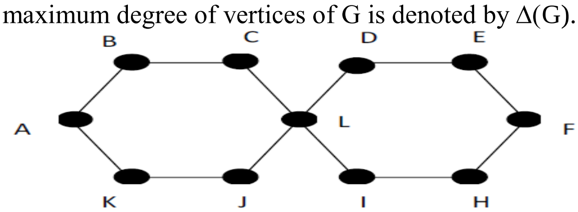


Multi Graph: A graph which has multiple edges but no loops is called a multigraph.



General Graph: A graph which contains more than couple of multiple edges or loops (or both) is called a general graph.

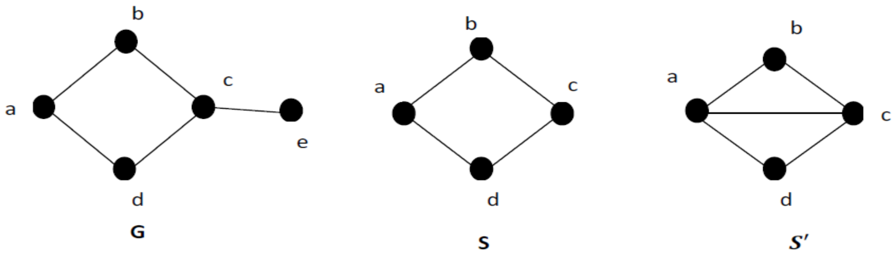
Degree of Vertex: Let G is the graph with sequential loops, and let v be a vertex of G. The designation of v is the number of edges meeting at v, and is denoted by deg (v). The minimum degree of vertices of G is denoted by $\delta(G)$ and the maximum degree of vertices of G is denoted by $\Delta(G)$.



Deg (A) = 2, Deg (L) = 4, Deg (H) = 2.

$(G) = 2$ & $\Delta(G) = 4$

Sub Graph: A sub graph S of a graph G is a graph so that the common vertices of S are a subset to the vertices of G. (i.e.) $V(S) \subseteq V(G)$ on edges of S are a subset to the edges of G.(i.e.)$E(S) \subseteq E(G)$.

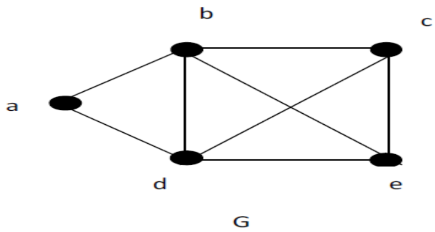


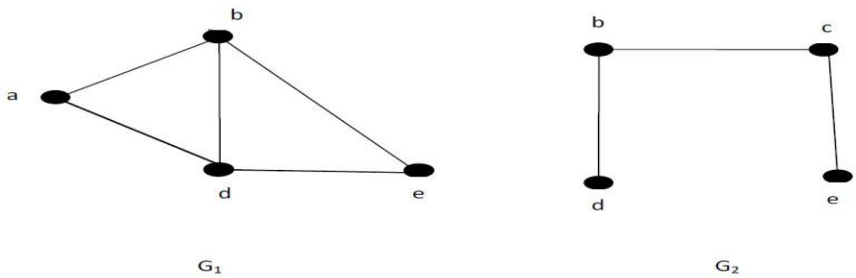
S is a subgraph of G

S^1 is not a subgraph of G

Induced Subgraph: A vertex commonly induced subgraph is one that consists of some activity of the vertices of the live graph and all of the edges that connect them in the originally denoted by ⟨V⟩.

Example:





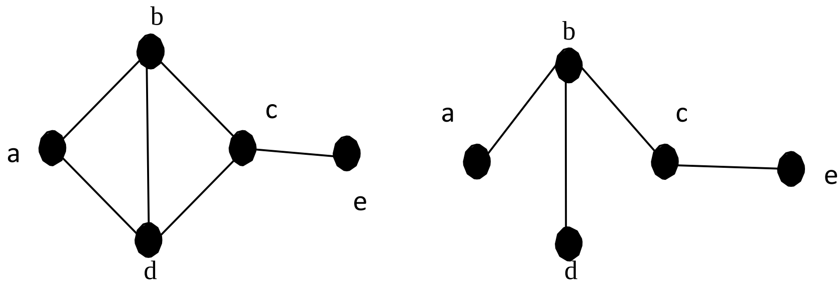
G_1 is an induced primal subgraph - induced by the set of vertices $V_1 = \{a, b, d, e\}$.

G_2 is not an induced vertices subgraph.

Proper Subgraph: If S is a subgraph of G then we denote like $S \subseteq G$. When $S \subseteq G$ but $S \neq G$.

Spanning Subgraph: A spanning subgraph of G is a subgraph that contains all the common vertices of G. (i.e.) V(S) = V(G). S is spanning subgraph of G.

Example:



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Triple Connected Domination Number of a Graph:

Definition: A dominating mutual set S of a connected via graph G is said to be a tripleconnected dominating set of induced G if the induced sub graph < S > is tripleconnected.

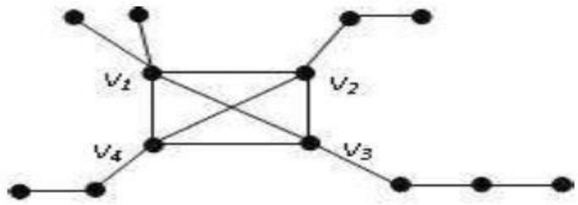
The minimum cardinality chosen over all triple connected dominating sets is the triple connected induced domination number and is denoted by (G).

Theorem: A tree T is triple connected if and only if $T \cong P_p; P \geq 3$.

Theorem: A connected graph G is not a triple connected if it is and only if there exists a H-cut with ω(G−H) ≥ 3 such that $|V(H) \cap N(C_i)| = 1$ for at least minimum three components C1, C2 and C3 of G −H.

Let G be a connected graph with m vertices $v_1, v_2, ..., v_m$. The mutual graph obtained from G by attaching combined n1 times a pendant vertex of P_{l1} on the vertex v1, n2 times a pendant pretend vertex of Pl2 on the vertex v2 and so on, is denoted by G(n1Pl1, n2Pl2 , n3Pl3 , . . . , nmPlm) where ni, li ≥ 0 and 1 ≤ i ≤ m.

Example: Let v1, v2, v3, v4 be the common vertices of K4. The graph K4(2P2, P3, P4, P3) is obtained from K4 by dully attaching 2 times a pendant vertex of P2 on v1,1timea pendant vertex of P3 on v2,1time a mutually pendant vertex of P4 on v3 and1time a pendant vertex of P3 on v4.



Paired Triple Connected Domination Number of a Graph:

Definition: A subset S of V of an on commonly an nontrivial graph G is said to be a strong triple connected dominating set, if S is a much strong dominating set and the induced subgraph <S> is a dominated triple connected. The minimum cardinality taken over all strong triple connected combined dominating sets is called the strong triple connected domination number of G also it's denoted by γ_{stc}(G). Any strong triple connected dominating set with γ_{stc} vertices is called a γ_{stc}-set of G.

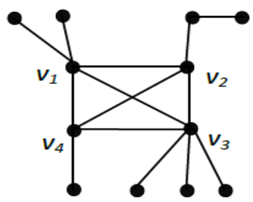
Example: For the graph C5 = v1v2v3v4v5v1, S = {v1, v2, v3, v4} forms a paired connected dominating set. Hence γ_{ptc}(C5)=4.

Theorem: G is semi-complete graph with p ≥ 4 vertices. Then G has a vertex of degree 2 if and only if one of the vertices of G has consequent neighborhood number p –3.

Theorem: G is semi-complete graph with p≥4 vertices such that there is a vertex with consequent neighbourhood number p – 3. Then γ(G) ≤2. Let G be a connected graph with m vertices $v1, v2, v2, \ldots \ldots$,. The graph $G(n1Pl1,\ n2Pl2,\ n3Pl3,\ \ldots\ \ldots,\ nmPlm)$, where *ni, li* ≥ 0 and 0≤i≤m, is obtained from G by pasting n1 times a pendant vertex of P_{l_1} on the vertex v1,n2 times a pendant vertex of P_{l_2} on the vertex v1 and soon.

Example: Let *v1, v2, v3, v4,* be the vertices of *K4*, the graph *K4(2P2,P3,3P2,P2)* is obtained from *K4* by pasting 2 times a pendant vertex of *P2* on *v1*, 1 times a pendant vertex of *P3* on *v2*, 3 times a pendant vertex of *P2* on *v3* and 1 times a pendant vertex of *P2* on *v4* and the graph in*G1*.

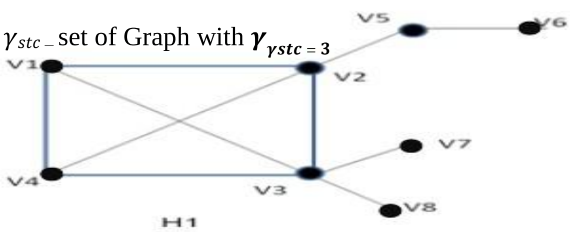
G1:



Strong Triple Connected Domination Number of a Graph:

Definition: A subset S of V of an on in trivial graph G is said to be a strong triple well connected dominating set, if S is a strong dominating set then the induced subgraph <S> is a triple connected. The minimum cardinality taken overall strong triple connected dominating sets is called the strong triple connected domination number of G and is denoted by γ_s(G). Any strong triple connected dominating set with γ_{stc} vertices is called a γ_{stc}-set of G.

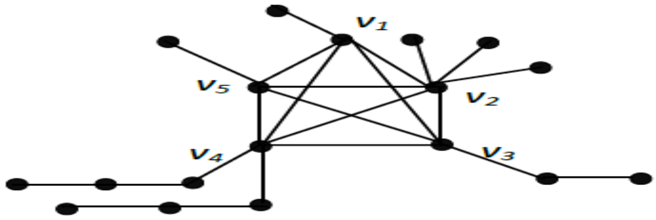
Example: For the graph H1, S = {v1, v2, v3} forms a γ_{stc} – set of Graph with $\boldsymbol{\gamma_{ystc=3}}$



Theorem: Let G be any graph and D be any dominating set of G. Then $|V - D| \leq \sum_{u \in V(D)} \deg(u)$ and equality hold in this relation if and only if D has the following properties.

- ✓ D is independent
- ✓ For every $u \in V - D$, there exists a unique vertex $v \in D$ such that $N(u) \cap D = \{v\}$.

Example: Let v_1, v_2, v_3, v_4, be the vertices of K_5. The graph $K_5(P_2, 3P_2, P_3, 2P_4, P_2)$ is obtained from K_5 by attaching 1 time a pendant vertex of P_2 on v_1, 3 time a pendant vertex of P_2 on v_2, 1 time a pendant vertex of P_3 on v_3 and 2 times a pendant vertex of P_4 on v_4, 1 time a pendant vertex of P_2.

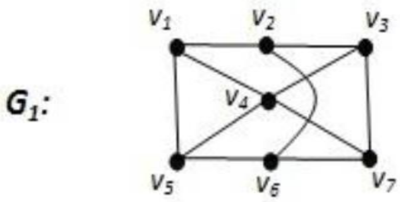


K_5 (P_2, $3P_2$, P_3, $2P_4$,P_2)

Weak Triple Connected Domination Number of a Graph:

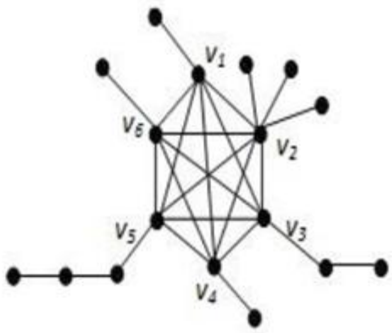
Definition: A subset S of V of an on in travel graph G is said to be a less strengthen triple bitted dominating set, if S is a low dominating set and the acclimated subgraph <S> is triple connected. The nominal cardinality taken over all weak triple bitted dominating sets is called the low triple connected domination number of G and it's knower by $\gamma_{wtc}(G)$. Any weak triple connected dominating set with γ_{wtc} vertices is called a γ_{wtc} set of G.

Example: For the graph G_1, S = {v_1, v_2, v_3} forms a γ_{wtc}-set of G. Hence $\gamma_{wtc}(G_1) = 3$.



Let G is a connected graph conditionally with m vertices v_1, v_2, …., v_m. The graph obtained from G by attaching n_1 lots of times a pendant vertex of P_{l_1} with conducted on the vertex v_1, n_2 times a pendant vertex of P_{l_2} on the vertex v_2 and so on, is denoted by $G(n_1P_{l_1}, n_2P_{l_2}, n_3P_{l_3}, …., n_mP_{l_m})$ where n_i, $l_i \geq 0$ and $1 \leq i \leq m$.

Example: Let v_1, v_2, v_3, v_4, v_5, v_6 be the vertices of K_6. The graph K_6 (P_2, $3P_2$, P_3, P_2, P_4, P_2) is obtained from K_6 by attaching 1 time a collided pendant vertex of P_2 on v_1, 3 time a pendant vertex of P_2 on v_2, 1 time a pendant vertex of P_3 on v_3 and 1 time a pendant vertex of P_2 on v_4, 1 time a pendant vertex of P_4 on v_5, 1 time of a pendant vertex rurally of P_2 on v_6.



Observation: Weak triple connected dominating set (wtcd set) does not exists forall graphs and if exists, then $\gamma_{wtc}(G) \geq 3$.

Example: For the graph G_2, any minimum triple connected dominating setmust contain the v_5 and any triple connected dominating set containing v_5 is nota weak triple connected and hence γ_{wtc} does not exists.

Observation: The complement of a weak triple connected dominating set crucially need notbe a weak triple connected dominating set.

Observation: Always every weak triple connected dominating set is a triple dominating combined setbut notconversely.

Observation: Every weak triple connected dominating set is a dominating set butnot seriously conversely.

Conclusion:

The concept of triple connected digraphs and domination in triple connected digraphs can be applied to physical problems such as flow networks with valves in the pipes and electrical networks, neural networks etc. They were applied in abstract representations of introduction of computer programs and are an invaluable tools in the study of sequential machines. In future this paper can be extended to studies of strong and weak domination in triple connected digraphs.

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