International Journal of Engineering Research and Modern Education (IJERME)
ISSN (Online): 2455 - 4200
(www.rdmodernresearch.com) Volume I, Issue I, 2016

ANALYTICAL EXPRESSIONS OF THE THERMAL BOUNDARY-LAYER OVER A FLAT PLATE WITH A CONVECTIVE SURFACE BOUNDARY CONDITION

Dr. C. Thangapandi*, Dr. V. Ananthaswamy* & S. Punitha**
* Associate Professor, Department of Mathematics, The Madura College, Madurai, Tamil Nadu
** Assistant Professor, Department of Mathematics, The Madura College, Madurai, Tamil Nadu
** M.Phil Scholar, Department of Mathematics, The Madura College, Madurai, Tamil Nadu

Abstract:

In this research paper, the modified Homotopy analysis methods are adopted to study the problem of thermal boundary-layer over a flat plate with a convective surface boundary condition analytically. The analytical expressions of the dimensionless velocity and temperature profile for the system of non-linear differential equations with infinite boundary conditions are derived using the modified Homotopy analysis method. Further the graphical representations of the dimensionless velocity and temperature profiles are investigated. The Homotopy analysis method can be easily extended to solve other non-linear initial and boundary value problems in physical, chemical and biological sciences.

Keywords: Thermal Boundary Layer, Non-Linear Differential Equation, Convective Boundary Condition & Modified Homotopy Analysis Method.

1. Introduction:

Most of the problem and scientific phenomena such as physical problems, fluid mechanics, engineering application and chemical physics it can be described through non-linear equations. Except the limited number of problems most of them do not have an exact solution. This method is applied to solve the linear and non-linear problems. Nowadays, many authors have the attention to study the solution of non-linear differential equation by using various methods, such as Homotopy perturbation method (HPM), Homotopy analysis method (HAM), Differential transform method (DTM), these methods are used to reduce the equation and we obtain the exact or approximate solution. The concept of Homotopy analysis method was first introduced by Liao in 1992 [1-4] is method offers highly accurate successive approximations of the solution. Howarth [5] developed this method for the solution of the laminar boundary-layer equation they applied the method of LTNHPM, Chen et.al [6] and [7] developed this method for partial differential equations and obtained closed form series solution for linear and non-linear initial value problems.

In the present work, the analytical expressions of the thermal boundary layer over a flat plate with a convective surface boundary condition are derived by using modified Homotopy analysis method. The Homotopy analysis method (HAM) is a general analytical method to solve strongly non-linear differential equation. In addition, many authors have investigated the problem of thermal boundary-layer and find the similarity solution for the case of constant surface temperature at the plate. In this problem we consider the value based on the boundary conditions, we guess our initial solutions for the dimensionless temperature and the dimensionless stream function.

2. Mathematical formulation of the problem:

Consider the two dimensional thermal boundary layer flow with a convective surface boundary condition with the semi-infinite flat plate in the cold fluid at temperature T_∞ its moves over the top surface of the plate with the uniform velocity U_∞.For the steady two dimensional flow, the equation of continuity, momentum, and energy equations can be written as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

Where u is the velocity components in x and v is the velocity components in y direction of the fluid, T denotes the fluid temperature, ν denotes the kinematic viscosity of the fluid and α denotes the thermal diffusivity of the fluid. The velocity boundary condition can be expressed as

$$u(x,0)=v(x,0)=0. \tag{4}$$

$$u(x,0)=U_\infty \tag{5}$$

The bottom surface of the plate is heated by convection from a hot fluid at temperature T_f which provides a heat transfer coefficient h_f. The boundary conditions at the plate surface and far into the cold fluid may be written as

$$-k\frac{\partial T}{\partial y}(x,0)=h_f[T_f-T(x,0)]. \tag{6}$$

$$T(x,0)=T_\infty \tag{7}$$

We may assume that incompressible flow and constant properties then, the momentum equation is decoupled from the energy equation.

Let x,y,u,v,T be the dimensionless in terms of arbitrarily selected reference length x_0 the characteristic velocity is referred as U_∞ and characteristic temperature is referred as T_∞ respectively, with replacing dimensionless variable and the similarity solution method, we have the following momentum equation.

$$2f'''+ff''=0, \tag{8}$$

$$\eta=0: f(0)=0,\ f'(0)=0 \tag{9}$$

$$\eta\to\infty:\ f'(\infty)=1 \tag{10}$$

Where

$$\eta=y\sqrt{\frac{U_\infty}{\nu x}} \tag{11}$$

$$\Psi=f(\eta)U_\infty\sqrt{\nu x/U_\infty}\,. \tag{12}$$

Hence

$$u=U_\infty f'(\eta),\qquad v=\frac{1}{2}\frac{\sqrt{\nu U_\infty}}{x}(\eta f'-f). \tag{13}$$

$$\frac{d^2T}{d\eta^2}+\frac{1}{2}prf\frac{dT}{d\eta}=0 \tag{14}$$

Here *Pr* is denoted by Prandtl number, which is equal to the ratio of the momentum diffusivity of the fluid to its thermal diffusivity $\left(i.e.\ \Pr=\nu/\alpha\right)$.

A dimensionless temperature parameter is defined as follows

$$\theta = \frac{T - T_\infty}{T_f - T_\infty} \tag{15}$$

Hence, the boundary –layer energy equation then becomes

$$\theta'' + \frac{1}{2}\Pr f\theta' = 0. \tag{16}$$

$$\theta'(0) = -a[1 - \theta(0)], \theta(\infty) = 0. \tag{17}$$

Where $$a = \frac{h_1}{k}\sqrt{\nu x / U_\infty} \tag{18}$$

For the energy equation to have a similarity solutions, the quantity 'a' must be a constant and not a function of x as in eqns. (18). This condition can be met if the heat transfer coefficient h_f is proportional to $x^{-1/2}$. We therefore assume

$$h_f = cx^{-\frac{1}{2}} \tag{19}$$

where c is a constant.
With the introduction of the eqn. (19) into an eqn. (18), we have

$$a = \frac{c}{k}\sqrt{\nu / U_\infty} \tag{20}$$

In the next section we shall solve the system of eqns. (8) and (16) by using the HAM. The equations are coupled and non-linear.

3. Solution of the Non-Linear Boundary Value Problem using the Modified Homotopy Analysis Method:

HAM is a non-perturbative analytical method for obtaining series solutions to nonlinear equations and has been successfully applied to numerous problems in science and engineering [8-27]. In comparison with other perturbative and non perturbative analytical methods, HAM offers the ability to adjust and control the convergence of a solution via the so-called convergence-control parameter. Because of this, HAM has proved to be the most effective method for obtaining analytical solutions to highly non-linear differential equations. Previous applications of HAM have mainly focused on non-linear differential equations in which the non-linearity is a polynomial in terms of the unknown function and its derivatives. As seen in (1), the non-linearity present in electrohydrodynamic flow takes the form of a rational function, and thus, poses a greater challenge with respect to finding approximate solutions analytically. Our results show that even in this case, HAM yields excellent results.

Liao [8-13] proposed a powerful analytical method for non-linear problems, namely the Homotopy analysis method. This method provides an analytical solution in terms of an infinite power series. However, there is a practical need to evaluate this solution and to obtain numerical values from the infinite power series. In order to investigate the accuracy of the Homotopy analysis method (HAM) solution with a finite number of terms, the system of differential equations were solved. The Homotopy analysis method is a good technique comparing to another perturbation method. The Homotopy analysis method contains the auxiliary parameter h, which provides us with a simple way to adjust and control the convergence region of solution series. Using this method, we can obtain the following solution to (8) and (16) (see Appendix B). The approximate analytical solution of the equations (8) and (16) by using HAM are given by

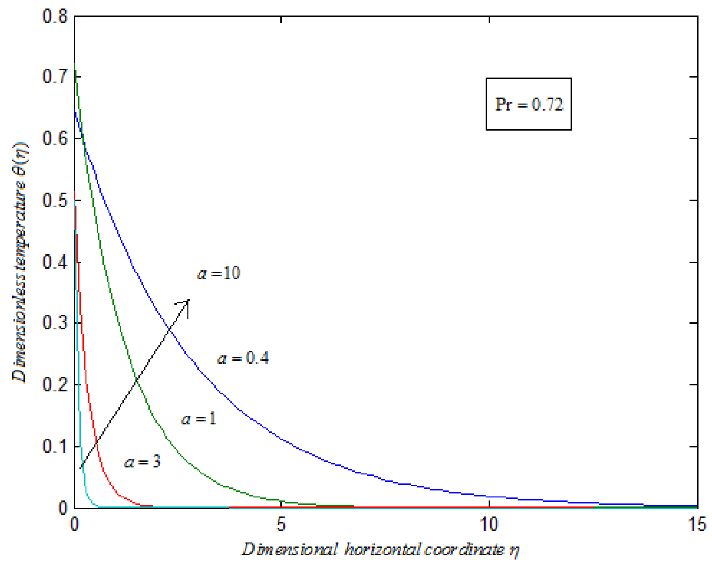
$$f(\eta) = (\eta - 1 + e^{-\eta}) - h\left(\frac{1}{2}\left(\eta e^{-\eta} + 2e^{-\eta} + \frac{e^{-2\eta}}{8}\right) + \frac{5\eta}{8} - \frac{17}{16}\right) \quad (21)$$

$$\theta(\eta) = \frac{1}{2}e^{-\eta a} - h\frac{a}{4}\Pr\left(\frac{\eta e^{-a\eta}}{a^2} + \frac{2e^{-a\eta}}{a^3} - \frac{e^{-a\eta}}{a^2} + \frac{e^{-\eta(a+1)}}{(a+1)^2}\right) \quad (22)$$

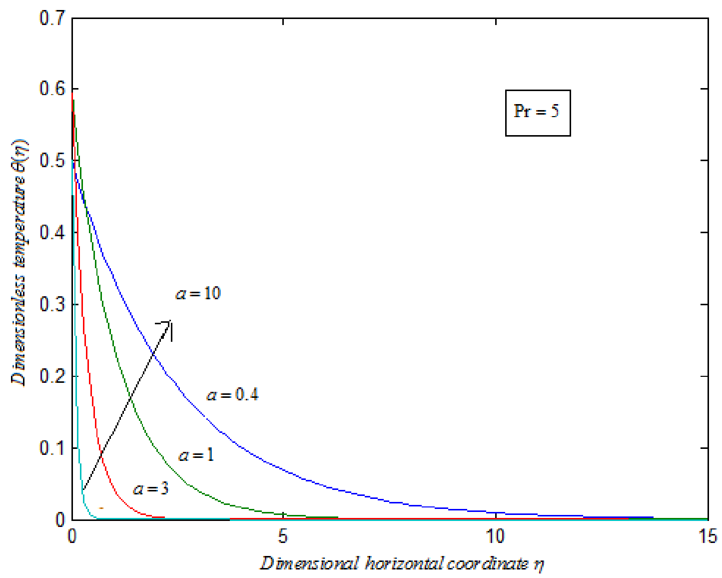
4. Results and Discussion:

The main interest in this section is to investigate the effects of Prandtl number *Pr* and the dimensionless parameter a using Modified Homotopy analysis method. Figure.1 shows the dimensional horizontal coordinate η versus the dimensionless temperature $\theta(\eta)$, when the parameter a value increases, the dimensionless temperature $\theta(\eta)$ decreases for various values of the other dimensionless parameter *Pr*. When the parameter a increases then the flat plate surface dimensionless temperature decreases in all the Figs. In Table.1 the numerical values of $f(\eta), f'(\eta), f''(\eta)$ are obtained by using the Modified Homotopy analysis method.

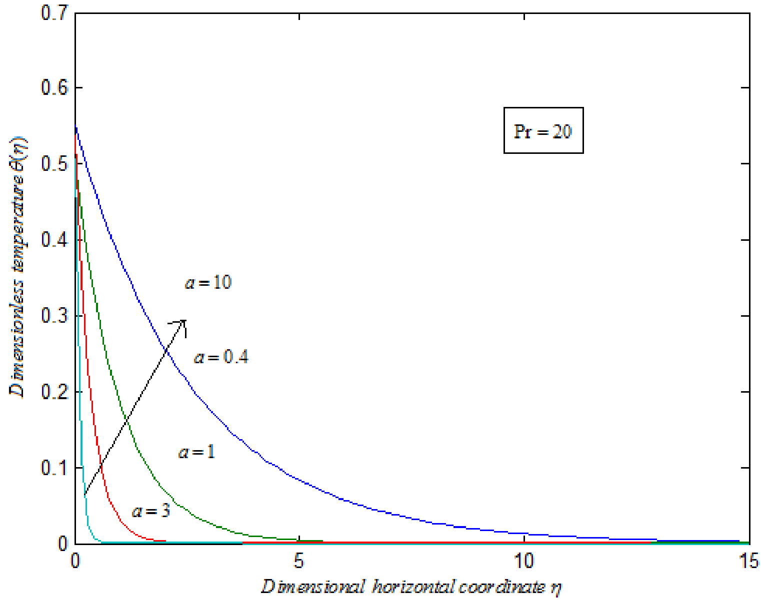
(a)



(b)



(c)



(d)

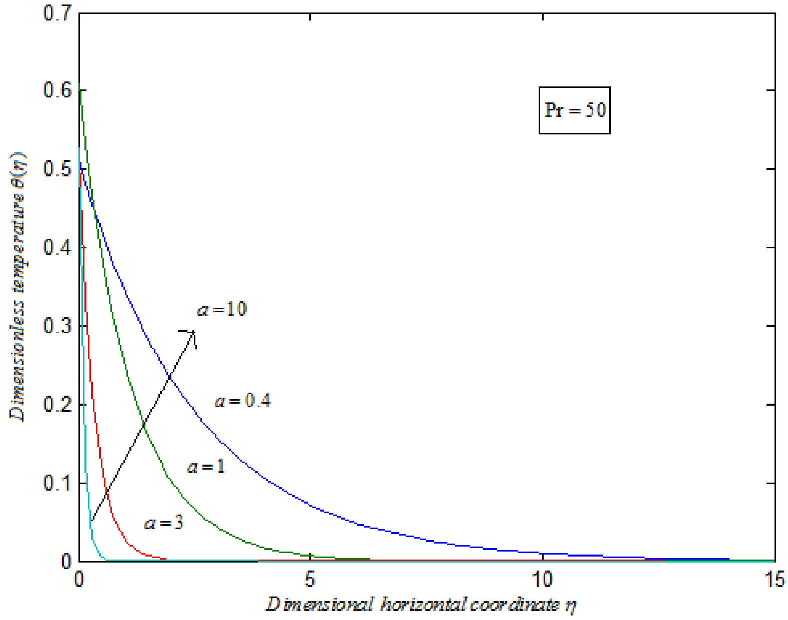


Fig 1: Dimensional horizontal coordinate η versus the dimensionless temperature $\theta(\eta)$. The curves are plotted using the eqn. (22) for various values of the the dimensionless parameter a and in some fixed values of the other dimensionless parameter *Pr*, when (a) *Pr=0.72*, (b) *Pr=5*, (c) *Pr=20*, (d) *Pr=50 and* $h=-0.3725$.

Table 1: The numerical values for $f(\eta)$, $f'(\eta)$, $f''(\eta)$ are obtained from eqn. (21) by using the modified HAM

η	$f(\eta)$	$f'(\eta)$	$f''(\eta)$
	$h=0.6$	$h=0.3$	$h=-0.8$
0.0	0.00000	0.00000	0.33250
1	0.16529	0.31985	0.32388
2	0 .65167	0.62962	0.26685
3	1.39380	0.84518	0.16198

4	2.30193	0.95272	0.64282
5	3.25574	0.99386	0.01584
6	4.27417	0.99813	0.00248
7	5.27125	0.99970	0.00020
8	6.26370	1.00028	0.00001
9	7.27001	1.00050	0.00001

5. Conclusions:

The analytical expressions of the dimensionless stream function $f(\eta)$ and the dimensionless temperature $\theta(\eta)$ in the thermal boundary layer over a flat plate with a convective surface boundary condition are derived by using the modified HAM for all values of dimensionless parameters a and *Pr.* The numerical values of $f(\eta), f'(\eta), and\ f''(\eta)$ results are listed in Table.1 .The HAM and modified HAM requires less computational work than existing approaches while supplying quantitatively reliable results. The results show us the validity and great potential of the modified HAM for heat and mass transfer problems in engineering and applied sciences.

Acknowledgement:

Researchers express their gratitude to the Secretary Shri. S. Natanagopal, Madura College Board, Madurai, Dr. K. M. Rajasekaran, The Principal and Dr. S. Muthukumar, Head of the Department, Department of Mathematics, The Madura College, Madurai, Tamilnadu, India for their constant support and encouragement.

References:

1. S. J. Liao, The proposed Homotopy analysis technique for the solution of non linear problems, Ph.D. Thesis, Shanghai Jiao Tong University, (1992).
2. S. J. Liao, An approximate solution technique which does not depend upon small parameters: a special example, Int. J. Non-Linear Mech. 30 (1995): 371-380.
3. S. J. Liao, Beyond perturbation introduction to the Homotopy analysis method, 1st edn. Chapman and Hall, CRC press, Boca Raton, 67 (2003): 336.
4. Y.Y.Wu, S.J.Liao, Solving the one loop solution of the Vakhnenko equation by means of the Homotopy analysis method, Chaos Solit.Frac. 23 (2004): 1733-1740.
5. L. Howarth, On the solution of the laminar boundary-layer equations, Proc. Roy. Sot. London, A164 (1938): 547.
6. C.K. Chen, S.H. Ho, Solving partial differential equations by two dimensional differential transform method, Appl. Math. Comput., 106 (1999): 171–179
7. M.M.Rashidi, Analytical Solution of the Thermal Boundary-Layer Over a Flat Plate with a Convective surface boundary condition, International conference on signal processing system, INSPEC : 10789195 (2009): 905-909.
8. S. J. Liao, On the Homotopy analysis method for nonlinear problems, Appl. Math. Compute. 147 (2004): 499-513.
9. S. J. Liao, An optimal Homotopy-analysis approach for strongly nonlinear differential equations, Commun. Nonlinear Sci. Numer. Simulat., 15 (2010): 2003-2016.
10. S. J. Liao, The Homotopy analysis method in non linear differential equations, Springer and Higher education press, 2012.
11. S. J. Liao, An explicit totally analytic approximation of Blasius viscous flow problems. Int J. Nonlinear Mech., 34 (1999): 759–778.
12. S. J. Liao, On the analytic solution of magnetohydrodynamic flows non-Newtonian fluids over a stretching sheet. J Fluid Mech: 488 (2003): 189–212.

13. S. J. Liao, A new branch of boundary layer flows over a permeable stretching plate. Int J Nonlinear Mech, 42 (2007): 819–830.
14. G. Domairry, H. Bararnia, An approximation of the analytical solution of some nonlinear heat transfer equations: a survey by using Homotopy analysis method, Adv. Studies Theor. Phys., 2 (2008): 507-518.
15. Y. Tan, H. Xu, S.J. Liao. Explicit series solution of travelling waves with a front of Fisher equation. Chaos Solitons Fractals: 31 (2007): 462–472.
16. S.Abbasbandy, Soliton solutions for the FitzhughNagumo equation with the Homotopy analysis method. Applied Mathematical Modeling, 32 (2008): 2706–2714.
17. J.Cheng , S.J.Liao, R.N.Mohapatra, K.Vajravelu, Series solutions of nano boundary layer flows by means of the Homotopy analysis method. J Math Anal Appl., 343 (2008): 233– 245.
18. T.Hayat, Z.Abbas , Heat transfer analysis on MHD flow of a second grade fluid in a channel with porous medium. Chaos Solitons Fractal, 38 (2008): 556–567.
19. T.Hayat,R.Naz, M.Sajid, On the homotopy solution for Poiseuille flow of a fourth grade fluid. Commun Nonlinear Sci Numer Simul, 15 (2010): 581–589.
20. H. Jafari, C. Chun, S. M. Saeidy, Analytical solution for nonlinear gas dynamic Homotopy analysis method, Appl. Math., 4 (2009): 149-154.
21. M. Fooladi, S.R.Abaspour, A.Kimiaeifar and M. Rahimpour, On the analytical solution of nonlinear normal mode for continuous systems by means of HAM. World Applied Sciences Journal, 6 (2009): 297-302.
22. K.Saravanakumar, V.Ananthaswamy, M.Subha, and L.Rajendran, Analytical Solution of nonlinear boundary value problem for in efficiency of convective straight Fins withtemperature-dependent thermal conductivity, ISRN Thermodynamics, Article ID 282481(2013): 1-18.
23. M.Subha, V.Ananthaswamy and L.Rajendran, Analytical solution of non-linear boundary value problem for the electrohydrodynamic flow equation, International Jouranal of Automation and Control Engineering, 3(2), (2014): 48-56.
24. V. Ananthaswamy, M. Subha, Analytical expressions for exothermic explosions in a slab, 123International Journal of Research – Granthaalayah, 1(2), (2014): 22-37.
25. V. Ananthaswamy, S. Uma Maheswari, Analytical expression for the hydrodynamic fluid flow through a porous medium, International Journal of Automation and Control Engineering, 4(2), (2015): 67-76.
26. V. Ananthaswamy, L. Sahaya Amalraj, Thermal stability analysis of reactive hydromagnetic third-grade fluid using Homotopy analysis method, International Journal of Modern Mathematical Sciences, 14 (1), (2016): 25-41.
27. M. Sajid ,T. Hayat Comparison of HAM and HPM solutions in heat radiation equations. International Communications in Heat and Mass Transfer, 36(2009): 59-62.

Appendix: A

Basic Concept of the Homotopy Analysis Method [8-27]

Consider the following differential equation:

$$N[u(t)]=0 \tag{A.1}$$

Where N is a nonlinear operator, t denotes an independent variable, $u\,(t)$ is an unknown function. For simplicity, we ignore all boundary or initial conditions, which can be treated in the similar way. By means of generalizing the conventional Homotopy method, Liao (2012) constructed the so-called zero-order deformation equation as:

$$(1-p)L[\varphi(t;p)-u_0(t)]=phH(t)N[\varphi(t;p)] \tag{A.2}$$

where $p \in [0,1]$ is the embedding parameter, $h \neq 0$ is a nonzero auxiliary parameter, $H(t) \neq 0$ is an auxiliary function, L an auxiliary linear operator, $u_0\ (t)$ is an initial guess of $u(t)$, $\varphi(t:p)$ is an unknown function. It is important to note that one has great freedom to choose auxiliary unknowns in HAM. Obviously, when $p=0$ and $p=1$, it holds:

$$\varphi(t;0)=u_0(t) \text{ and } \varphi(t;1)=u(t) \tag{A.3}$$

respectively. Thus, as p increases from 0 to 1, the solution $\varphi(t;p)$ varies from the initial guess $u_0(t)$ to the solution $u\ (t)$. Expanding $\varphi(t;p)$ in Taylor series with respect to p, we have:

$$\varphi(t;p)=u_0(t)+\sum_{m=1}^{+\infty}u_m(t)p^m \tag{A.4}$$

where $u_m(t)=\dfrac{1}{m!}\dfrac{\partial^m\varphi(t;p)}{\partial p^m}\Big|_{p=0}$ (A.5)

If the auxiliary linear operator, the initial guess, the auxiliary parameter h, and the auxiliary function are so properly chosen, the series (A.4) converges at $p=1$ then we have:

$$u(t)=u_0(t)+\sum_{m=1}^{+\infty}u_m(t). \tag{A.6}$$

Differentiating (A.2) for m times with respect to the embedding parameter p, and then setting $p = 0$ and finally dividing them by $m!$, we will have the so-called mth -order deformation equation as:

$$L[u_m-\chi_m u_{m-1}]=hH(t)\Re_m(\vec{u}_{m-1}) \tag{A.7}$$

where

$$\Re_m(\vec{u}_{m-1})=\frac{1}{(m-1)!}\frac{\partial^{m-1}N[\varphi(t;p)]}{\partial p^{m-1}} \tag{A.8}$$

and

$$\chi_m=\begin{cases}0, & m\leq 1,\\ 1, & m>1.\end{cases} \tag{A.9}$$

Applying L^{-1} on both side of equation (A7), we get

$$u_m(t)=\chi_m u_{m-1}(t)+hL^{-1}[H(t)\Re_m(\vec{u}_{m-1})] \tag{A10}$$

In this way, it is easily to obtain u_m for $m\geq 1$, at M^{th} order, we have

$$u(t)=\sum_{m=0}^{M}u_m(t) \tag{A.11}$$

When $M\to+\infty$, we get an accurate approximation of the original equation (A.1). For the convergence of the above method we refer the reader to Liao [3]. If equation (A.1) admits unique solution, then this method will produce the unique solution.

Appendix: B

Approximate analytical expression of the non-linear differential eqns. (8) and (16) using the Modified Homotopy Analysis Method:

We indicate how the eqns. (21) and (22) are derived in this paper. To find the solution of the eqns. (8) and (16) we construct the Homotopy are as follows:

$$(1-p)f''' = hp[f''' + \frac{1}{2}ff''] \quad \text{(B.1)}$$

$$(1-p)\theta'' = hp\left[\theta'' + \frac{1}{2}\Pr f\theta'\right] \quad \text{(B.2)}$$

The approximate analytical solution of the eqns. (8) and (16) are as follows:

$$f = f_0 + pf_1 + p^2 f_2 + \ldots \quad \text{(B.3)}$$

$$\theta = \theta_0 + p\theta_1 + p^2\theta_2 + \ldots \quad \text{(B.4)}$$

Substituting the eqn. (B.3) into an eqn.(B.1) and (B.4) into an eqn.(B.2) we get

$$(1-p)\left[\frac{d^3(f_0 + pf_1 + p^2 f_2 + \ldots)}{d\eta^3}\right] = hp\left[\frac{d^3(f_0 + pf_1 + p^2 f_2 + \ldots)}{d\eta^3} + \frac{1}{2}(f_0 + pf_1 + p^2 f_2 + \ldots)\left(\frac{d^2(f_0 + pf\ + p^2 f_2 + \ldots)}{d\eta^2}\right)\right] \quad \text{(B.5)}$$

$$(1-p)\left[\frac{d^2(\theta_0 + p\theta_1 + p^2 f_2 + \ldots)}{d\eta^2}\right] = hp\left[\frac{d^2(\theta_0 + p\theta_1 + p^2\theta_2 + \ldots)}{d\eta^2} + \frac{1}{2}\Pr(f_0 + pf_1 + p^2 f_2 + \ldots)\left(\frac{d(\theta_0 + p\theta_1 + p^2\theta_2 + \ldots)}{d\eta}\right)\right] \quad \text{(B.6)}$$

Comparing the coefficients of like powers of p in the eqns. (B.5) and (B.6) we get,

$$p^0 : \frac{d^3 f_0}{d\eta^3} = 0 \quad \text{(B.7)}$$

$$p^0 : \frac{d^2\theta_0}{d\eta^2} = 0 \quad \text{(B.8)}$$

$$p^1 : \frac{d^3 f_1}{d\eta^3} - \frac{d^3 f_0}{d\eta^3} = h\left(\frac{d^3 f_0}{d\eta^3} + \frac{1}{2} f_0\left(\frac{d^2 f_0}{d\eta^2}\right)\right) \quad \text{(B.9)}$$

$$p^1 : \frac{d^2\theta_1}{d\eta^2} - \frac{d^2\theta_0}{d\eta^2} = h\left(\frac{d^2\theta_0}{d\eta^2} + \frac{1}{2}\Pr f_0\left(\frac{d\theta_0}{d\eta}\right)\right) \quad \text{(B.10)}$$

The initial approximations are as follows:

$$f_0(0) = 0,\ f_0'(0) = 0,\ f_0'(\infty) = 1\ ;\theta_0(\infty) = 0,\ \theta_0'(0) = -a(1-\theta(0)). \quad \text{(B.11)}$$

$$f_i(0) = 0,\ f_i'(0) = 0,\ f_i'(\infty) = 0,\ \theta_i(\infty) = 0, \theta_i'(0) = 0, i = 1,2,3,\ldots \quad \text{(B.12)}$$

Solving the eqns. (B.7) - (B.10) and using the boundary condition (B.11) and (B.12) we obtain the following results:

For this HAM solution, we choose the initial guesses in the following form which satisfies the eqn. (B.11).

$$f_0 = \eta - 1 + e^{-\eta} \quad \text{(B.13)}$$

$$\theta_0 = \frac{1}{2}e^{-a\eta} \quad \text{(B.14)}$$

By solving the eqns. (B.9) and (B.10) using the eqn. (B.12), we can obtain the following results

$$f_1 = \frac{1}{2}\left(\eta e^{-\eta} + 2e^{-\eta} + \frac{e^{-2\eta}}{8}\right) + \frac{5\eta}{8} - \frac{17}{16} \qquad \text{(B.15)}$$

$$\theta_1 = -\frac{a}{4}\Pr\left(\frac{\eta e^{-a\eta}}{a^2} - \frac{2e^{-a\eta}}{a^3} + \frac{e^{-a\eta}}{a^2} - \frac{e^{-\eta(a+1)}}{(a+1)^2}\right) \qquad \text{(B.16)}$$

According to the HAM, we can conclude that

$$f = \lim_{p\to 1} f(\eta) = f_0 + f_1 \qquad \text{(B.17)}$$

$$\theta = \lim_{p\to 1} \theta(\eta) = \theta_0 + \theta_1 \qquad \text{(B.18)}$$

After putting the eqns. (B.13), (B.15) into an eqn. (B.17) and the eqns. (B.14), (B.16) into an eqn. (B.18), we obtain the solution in the text as given in the eqns. (21) and (22).

Appendix: C

Nomenclature:

Symbol	Meaning
U	Velocity components in the x direction
V	Velocity components in the y direction
Pr	Prandtl number
T	Temperature
X	Dimensional vertical coordinate
Y	Dimensional horizontal coordinate
T_∞	Local ambient temperature
U_∞	Uniform velocity
h_f	Heat transfer coefficient
T_f	Fluid temperature
U	Withdrawal velocity
α	Thermal diffusivity
η	Similarity variable
ψ	Radiation-conduction parameter
θ	Dimensionless temperature