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STOCHASTIC MODEL TO FIND THE AGE SPECIFIC CHD MORTALITY RATES IN MEN COMPARED WITH WOMEN USING UNIFORM DISTRIBUTION

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Abstract:

The gender difference (gender gap) in mortality due to coronary heart disease (CHD) decreases with age. This relationship has not been well characterized in diverse populations. The purpose of this study wad to examine the age specific coronary heart disease mortality rates in men compared with women through telegrapher's process with the help of uniform distribution.

Key Words: Coronary Heart Disease, Hormone Replacement Therapy, Telegrapher's Process & Uniform Distribution.

1. Introduction:

Although Coronary Heart Disease (CHD) is the leading cause of death in men and women, age-specific CHD mortality rates are strikingly higher in men compared with women. In general, both CHD incidence and mortality rates in women lag 10 years behind those of men [10]. It is well established that the gender difference is more pronounced at younger ages, such that 1 in 17 women has had a coronary event before age 60, in contrast with 1 in 5 men. The gender difference has been reported to decrease with age and after age 60, CHD accounts for 1 in 4 deaths in both sexes.

Previous United States based studies addressing gender differences in CHD mortality have been limited to predominantly Caucasian populations [7], [18] & [19]. Many studies have also examined black and white differences in CHD mortality, but none has directly compared the gender gap between ethnic groups [2], [5], [8], [20] & [21]. The gender gap in CHD mortality has been attributed to various factors. Differential prevalence and impact of traditional cardiovascular risk factors have been shown to account for part but not all of the gender difference.

Estrogen has been implicated as a possible protective factor in women because of an observed 2-fold increased CHD incidence in surgically postmenopausal vs premenopausal women of the same age. However, the use of hormone replacement therapy (HRT) has not been shown to reduce CHD events in postmenopausal women and the role of endogenous estrogen in the cardio protection of women compared with men is not completely understood. International data suggest that geography, secular trends, and environmental factors also play a role in gender differences in CHD occurrence [20]. The purpose of this study wad to examine the age specific coronary heart disease mortality rates in men compared with women through telegrapher's process with the help of uniform distribution.

In this paper we consider the two valued integrated telegraph signal with rightward velocity c_1 and leftward velocity $-c_2$ $(c_1, c_2 > 0)$ and rates λ_1, λ_2 of the occurrence of velocity switches. The classical case $(c_1 = c_2 = c; \lambda_1 = \lambda_2 = \lambda)$ has been studied in many papers and important probabilistic distributions and representations have been obtained independently by various authors and by different methods (for example [3], [4], [6] & [16]). When $c_1 \neq c_2$ and $\lambda_1 \neq \lambda_2$, the motion differs from that in the classical case in that it displays a drift whose components have also been studied (See [1], [14] & [15]). One component of the drift depends on the different velocities and the other on the different rates. These components differ substantially in the mathematical treatment they necessitate.

In particular, when $\lambda_1 \neq \lambda_2$, the elimination of the drift requires the Lorentz transformation of Special Relativity Theory. This was first noted by Cane [1] and further examined in [14] & [15] but nowhere has an accurate analysis of the transformation and its probabilistic implications been carried out. Here we discuss the random motion in the original frame of reference (x, t) and in the related relativistic one, (x', t') where the drift has been eliminated. The space coordinate x' must move with velocity $v_r = \frac{c_1 - c_2}{2} + \frac{(\lambda_2 - \lambda_1)(c_1 + c_2)}{2(\lambda_2 + \lambda_1)} = \frac{\lambda_2 c_1 - \lambda_1 c_2}{\lambda_1 + \lambda_2}$ with respect to the original frame of reference and the time t' must either be speeded up or slowed down with respect to t, in order to eliminate the drift. In the frame (x', t'), the particle moves with velocities $c' = \pm \frac{2(c_1 + c_2)\lambda_1 \lambda_2}{(\lambda_1 + \lambda_2)^2}$ initially chosen with equal probability 1/2, and the switches from positive to negative values and vice versa are governed by a homogeneous Poisson process with rate $\lambda' = \pm \frac{2\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}$

Therefore, the probabilist, in the reference (x', t') attributes to the random position of the particle, a symmetric distribution $p = p(x', t')$. Returning to the original coordinates and writing down the asymmetric distribution $p = p(x, t)$ requires careful attention due to the fact that here, differently from the Special Relativity theory, the adjustment of time depends on the random changes of the rates. In this paper we obtain the distribution $p = p(x, t)$ by means of the usual model, based on Fourier transforms.

2. Features of Motion and Governing Equation:

We assume that at time $t = 0$, a particle starts from the origin and that its initial velocity is the two valued random variable.

$$V(0) = \begin{cases} c_1 \text{ with probability } \frac{1}{2} \\ -c_2 \text{ with probability } \frac{1}{2} \end{cases}$$

where c_1, c_2 are positive, real numbers. The current velocity $V = V(t), t > 0$ switches from c_1 to $-c_2$ after an exponentially distributed time with parameter λ_1 and from $-c_2$ to c_1 after a random time with exponential distribution with parameter λ_2. The time intervals separated by velocity changes are independent random variables. Thus the particle moves forward with velocity c_1 and backward with velocity $-c_2$ and the changes are governed by a non homogeneous Poisson process. For the probabilistic description of the random position $X = X(t) = \int_0^t V(s)ds$ we need the following distributions

$$\begin{cases} f_1(x,t)dx = P_r\{X(t) \in dx, V(t) = c_1\} \\ f_2(x,t)dx = P_r\{X(t) \in dx, V(t) = -c_2\} \end{cases} \quad (1)$$

It is well known that the functions (1) are solutions of the following differential system (See [9])

$$\begin{cases} \frac{\partial f_1}{\partial t} = -c_1 \frac{\partial f_1}{\partial x} + \lambda_2 f_2 - \lambda_1 f_1 \\ \frac{\partial f_2}{\partial t} = c_2 \frac{\partial f_2}{\partial x} + \lambda_1 f_1 - \lambda_2 f_2 \end{cases} \quad (2)$$

The system (2) by means of the transformation

$$p = f_1 + f_2, w = f_1 - f_2$$

can equivalently be written as

$$\begin{cases} \frac{\partial p}{\partial t} = -\frac{c_1 - c_2}{2}\frac{\partial p}{\partial x} - \frac{c_1 + c_2}{2}\frac{\partial w}{\partial x} \\ \frac{\partial w}{\partial t} = -\frac{c_1 + c_2}{2}\frac{\partial p}{\partial x} - \frac{c_1 - c_2}{2}\frac{\partial w}{\partial x} - (\lambda_1 - \lambda_2)p - (\lambda_1 + \lambda_2)w \end{cases} \quad (3)$$

The distribution $p(x,t)dx = P_r\{X(t) \in dx\}$ consists of a singular component concentrated in $x = c_1 t$ with probability $\frac{1}{2}e^{-\lambda_1 t}$ and in $x = -c_2 t$ with probability $\frac{1}{2}e^{-\lambda_2 t}$ and an absolutely continuous part spread over the interval $(-c_2 t, c_1 t)$. The absolutely continuous part of the distribution is a solution of the second order hyperbolic equation (extracted from the differential system (3) by means of subsequent differentiations and substitutions):

$$\frac{\partial^2 p}{\partial t^2} = c_1 c_2 \frac{\partial^2 p}{\partial x^2} + (c_2 - c_1)\frac{\partial^2 p}{\partial x \partial t} - (\lambda_1 + \lambda_2)\frac{\partial p}{\partial t} + \frac{1}{2}[(c_2 - c_1)(\lambda_1 + \lambda_2) - (\lambda_2 - \lambda_1)(c_1 + c_2)]\frac{\partial p}{\partial x} \quad (4)$$

The presence of $\frac{\partial p}{\partial x}$ and $\frac{\partial^2 p}{\partial x \partial t}$ in (4) is clearly related to the drift of motion. Equation (4), when $c_1 = c_2 = c$ and $\lambda_1 = \lambda_2 = \lambda$, reduces to the classical telegraph equation

$$\frac{\partial^2 p}{\partial t^2} = c^2 \frac{\partial^2 p}{\partial x^2} - 2\lambda \frac{\partial p}{\partial t}$$

3. Formation of Model by using Initial Value Problem:

The classical approach based on Fourier transforms permits us to obtain the characteristic function

$$F(\beta, t) = \int_{-\infty}^{+\infty} e^{i\beta x}\, dP(x,t)$$

of the distribution

$$P(x,t) = P\{X(t) \le x\} \quad (5)$$

The characteristic function of the distribution (5) is

$$F(\beta, t) = \frac{1}{2}e^{-\frac{1}{2}\{-i\beta(c_2 - c_1) + (\lambda_1 + \lambda_2)\}t}[(1 + E_1)E_2 + (1 - E_1)E_3] \quad (6)$$

where $E_1 = \frac{\lambda_1 + \lambda_2}{\sqrt{(\lambda_1 + \lambda_2)^2 - \beta^2(c_1 + c_2)^2 + 2i\beta(c_1 + c_2)(\lambda_2 - \lambda_1)}}$

$$E_2 = e^{\frac{t}{2}\sqrt{(\lambda_1 + \lambda_2)^2 - \beta^2(c_1 + c_2)^2 + 2i\beta(c_2 + c_1)(\lambda_2 - \lambda_1)}}$$

$$E_3 = e^{-\frac{t}{2}\sqrt{(\lambda_1 + \lambda_2)^2 - \beta^2(c_1 + c_2)^2 + 2i\beta(c_2 + c_1)(\lambda_2 - \lambda_1)}}$$

for $\beta \in R$ and $t \ge 0$.

We first note that the Fourier transform of equation (4) is

$$\frac{d^2 F}{dt^2} + \{i\beta(c_2 - c_1)(\lambda_2 + \lambda_1)\}\frac{dF}{dt} + \left\{\frac{i\beta}{2}[(c_2 - c_1)(\lambda_2 + \lambda_1) - (\lambda_2 - \lambda_1)(c_2 + c_1) + \beta^2 c_1 c_2]\right\} = 0 \quad (7)$$

It is straightforward that the general solution of (7) reads

$$F(\beta, t) = e^{-\frac{t}{2}[-i\beta(c_2 - c_1) + (\lambda_1 + \lambda_2)]}[HE_2 + KE_3]$$

The constants H and K are evaluated using the fact that F must satisfies the following initial conditions:

$$F(\beta, 0) = 1$$
$$\frac{dF}{dt}(\beta, t) = \frac{1}{2}i\beta(c_2 - c_1) \text{ if } t = 0 \quad (8)$$

While the first condition immediately follows from the fact that $p(x,0) = \delta(x)$, the second one involves much more analysis. The features of motion described in above Section authorize us to write

$$Ee^{i\beta X(\Delta t)} = \frac{1}{2}e^{-ic_2 \Delta t}(1 - \lambda_2 \Delta t) + \frac{1}{2}e^{-ic_1 \Delta t}(1 - \lambda_1 \Delta t)$$

$$+\frac{(\lambda_1+\lambda_2)\Delta t}{(c_1+c_2)\Delta t}\frac{1}{2}\int_{-c_2\Delta t}^{c_1\Delta t} e^{i\beta y}\,dy + o(\Delta t) \quad (9)$$

since, in a small time elapse $[0,\Delta t)$, either no velocity change occurs or one Poisson event happens. From (9) and some calculations, we get

$$Ee^{i\beta X(\Delta t)} = 1 + \frac{i\beta}{2}(c_2 - c_1)\Delta t + o(\Delta t)$$

and thus $\frac{dF}{dt}|_{t=0} = \lim_{\Delta t\to 0}\frac{Ee^{i\beta X(\Delta t)}-1}{\Delta t} = \frac{i\beta}{2}(c_2 - c_1)$

as claimed in (8).

A little algebra permits us to calculate H and K and thus obtain (6). For the inversion of the characteristic function, we need three integrals which can be inferred from the relationship

$$\int_{-ct}^{ct} e^{i\gamma x} I_0\left(\frac{\lambda}{c}\sqrt{c^2t^2 - x^2}\right)dx = c\frac{\left(e^{\frac{t}{2}\sqrt{\lambda^2-c^2\gamma^2}} - e^{-\frac{t}{2}\sqrt{\lambda^2-c^2\gamma^2}}\right)}{\sqrt{\lambda^2-c^2\gamma^2}}$$

obtained in [17].

For the sake of simplicity, we write

$$A = \sqrt{(\lambda_1 + \lambda_2)^2 - \beta^2(c_1 + c_2)^2 + 2i\beta(c_2 + c_1)(\lambda_2 - \lambda_1)}$$

The formulas we must apply are

$$= (c_1 + c_2)e^{i\beta(c_2-c_1)\frac{t}{2} - \frac{(\lambda_2-\lambda_1)(c_2+c_1)}{2(c_2+c_1)}t}\left(\frac{e^{\frac{t}{2}A} - e^{-\frac{t}{2}A}}{A}\right)$$

$$\int_{-c_2t}^{c_1t} e^{i\beta x}\frac{(\lambda_2-\lambda_1)}{(c_2+c_1)}x\frac{\partial I_0}{\partial t}\left(\frac{2\sqrt{\lambda_1\lambda_2}}{c_1+c_2}\sqrt{(x + c_2t)(c_1t - x)}\right)dx \quad (10)$$

The third formula we need is

$$= e^{i\beta c_1 t}e^{\frac{\lambda_2-\lambda_1}{c_2+c_1}c_1 t} - e^{-i\beta c_2 t}e^{-\frac{\lambda_2-\lambda_1}{c_2+c_1}c_2 t} \quad (11)$$

Formulas (10) and (11) are closely connected.

With this at hand, the characteristic function (6) can be written as:

$$F(\beta,t) = \frac{e^{-\frac{(\lambda_1+\lambda_2)t}{2}}}{2}\left\{e^{-\frac{i\beta(c_2-c_1)}{2}}\left(e^{\frac{t}{2}A} - e^{-\frac{t}{2}A}\right) + (\lambda_1 + \lambda_2)e^{-\frac{i\beta(c_2-c_1)}{2}}\left(\frac{e^{\frac{t}{2}A} - e^{-\frac{t}{2}A}}{A}\right)\right\} \quad (12)$$

Using (11) for the first term and (10) for the second one, immediately obtain the distribution. In view of this distribution, we have to integrate the absolutely continuous part and use formulas (10), (11) and (12) when $\beta = 0$. We get

$$P_r\{-c_2t < X(t) < c_1t\} = \frac{e^{-\frac{(\lambda_1+\lambda_2)t}{2}}}{2}$$

$$\left\{(c_2 + c_1)e^{\frac{t}{2}(\lambda_1+\lambda_2)} - \frac{c_2+c_1}{2}e^{\frac{(\lambda_2-\lambda_1)t}{2}} - \frac{c_2+c_1}{2}e^{-\frac{(\lambda_2-\lambda_1)t}{2}}\right\}$$

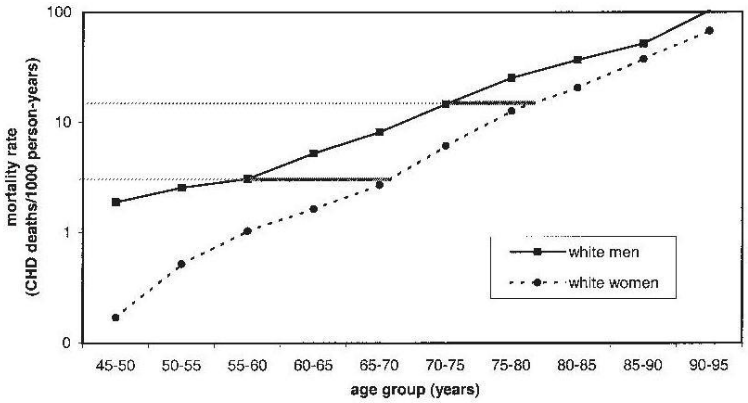
$$= 1 - \frac{1}{2}e^{-\lambda_1 t} - \frac{1}{2}e^{-\lambda_2 t} \quad (13)$$

4. Example:

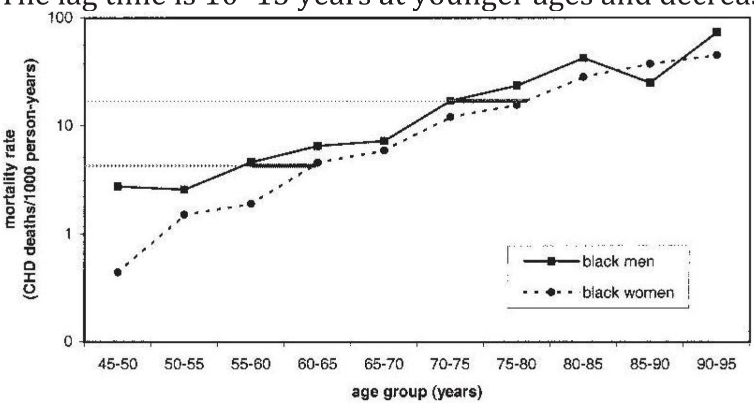
To examine the gender gap in CHD mortality across age groups and to compare the age dependency of the gender gap between blacks and whites, we conducted a prospective cohort study combining data from 9 United States epidemiological studies (Atherosclerosis Risk in Communities Study, Charleston Heart Study, Evans County Study, Framingham Heart Study [original and offspring cohorts], National Health Examination Follow-up Study, Rancho Bernardo Study, San Antonio Heart Study, and Tecumseh Community Health Study). Baseline examinations were performed between 1958 and 1990 (depending on the study), and mean follow up was 13.7 years in general communities in several United States geographic areas. We included 39,614 subjects >30 years and free of cardiovascular disease (CVD) at baseline (18% blacks, 37% men).

Completion of follow-up was >97% for all studies. As the main outcome measures, age specific CHD mortality rates and male/female CHD mortality hazard ratios were calculated using Cox hazards regression [11, 12 & 13].

Figure (1): Age Specific CHD mortality rates in men compared with women. The horizontal lines illustrate the lag times of CHD mortality rates between men and Women.

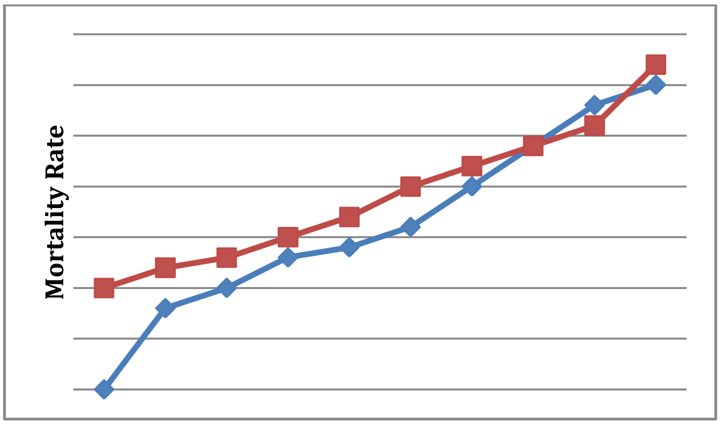


(A) Whites. The lag time is 10–15 years at younger ages and decreases with age

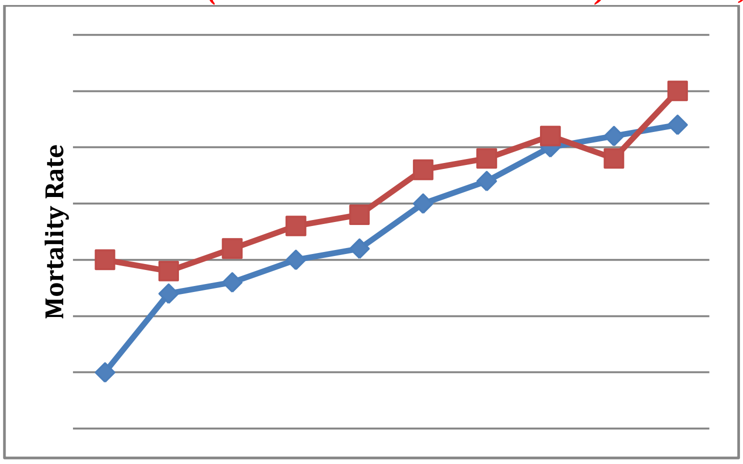


(B) Blacks. The lag time is 5–10 years

Figure (2): Age Specific CHD mortality rates in men compared with women. The horizontal lines illustrate the lag times of CHD mortality rates between men and women using Uniform Distribution.



(A) Whites. The lag time is 10–15 years at younger ages and decreases with age
Red Line: White Men
Blue Line: White Women



(B) Blacks. The lag time is 5–10 years
Red Line: Black Men
Blue Line: Black Women

5. Conclusion:

The gender difference in coronary heart disease mortality was more pronounced in whites than in blacks at younger ages. This discrepancy was not explained by adjustment for coronary heart disease risk factors and suggests that other factors may be responsible for the ethnic variation in the gender gap. By using uniform distribution the mathematical model gives the result as same as the medical report. The medical reports {Figure (1)} are beautifully fitted with the mathematical model {Figure (2)}; $(i.e)$ the results coincide with the mathematical and medical report.

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