



HAMILTON JACOBI BELLMANN EQUATION MODEL TO FIND THE INCRETIA ACTIVITY OF GLUCAGON LIKE PEPTIDE-1 WITH TYPE-2 DIABETES PATIENTS USING BOUNDARY CONDITIONS

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Abstract:

The aim is to evaluate in type-2 diabetes, the overall incretion effect is reduced. The present investigation was designed to compare insulinotropic actions of exogenous incretion hormones (GIP) and glucagons like peptide 1 (GLP -1) in nine type-2 diabetic patients and in nine age- and weight-matched normal subjects. Plasma GIP and GLP-1 concentrations were comparable to those after oral glucose with the low, and clearly supraphysiological with the high infusion rates. Both GIP and GLP-1 dose – dependently augmented insulin secretion in both groups. With GLP-1 type -2 diabetic patients reached 71% of the increments in C- peptide of normal subjects. In this paper, the problem is investigated by considering the boundary condition of Hamilton Jacobi-Bellmann equation.

Key Words: Glucagon Like Peptide-1 (GLP-1), Stochastic Model, Type-2 Diabetics, HJB Equation & Ornstein Uhlenbeck Process.

1. Introduction:

In normal subjects oral glucose enhances insulin secretion more than does intravenous glucose infusion. This augmentation of insulin secretion is due to the secretion and action of gut hormones [3] with insulinotropic activity, namely, gastric inhibitory polypeptide (GIP) from the upper gut and glucagons like peptide 1 (GLP-1) from the lower gut. In type-2 diabetic patients, the incretia effect is reduced or lost. The present study was designed to compare, the insulinotropic and glucagon - lowering actions of both synthetic human GIP and GLP-1, infused at both approximately physiological and pharmacological concentrations, in matched groups of type -2 diabetic patients and normal subjects.

In this paper the problem is investigated by using the boundary condition of Halmilton Jacobi Bellman equation [9]. The continuous time portfolio optimization problem in Kim and Omberg [10]. The sufficient conditions to verify that a solution derived from the Hamilton-Jacobi-Bellman equation are in fact an optimal solution to the portfolio selection problem. Many studies have been done on continuous-time portfolio optimization problem with the Merton's seminal work [6], [7] & [8]. In particular, there has been increasing interest in finding an optiomal portfolio strategy when investment opportunities are stochastic, because many empirical works conclude that investment opportunities are time-varying. There are two main approaches in solving continuous-time portfolio optimization problem. One is the stochastic control approach and the other is the martingale approach. In the stochastic control approach, an optimal solution is conjectured by guessing a solution to the HJB equation. It is

necessary to verify that the conjectured solution is in fact solution to the original problem. Korn and Kraft [6] pointed out, the verification is often skipped since it is mathematically demanding for Kim and Omberg examined the finiteness of conjectured value function very carefully, but they could not provide verification conditions. The sufficient condition to verify that the conjectured solution is in fact the solution to the original problem.

2. Stochastic Model:

Let (ψ, D, P) be a complete probability space on which we define a two-dimensional standard Brownian motion $B = (B^1, B^2)^T$ and we also fix a time interval $[0, T]$. Let $D(t)$ be the augmentation of the filtration $D^B(t) := \omega(B(a); 0 < a < t), 0 < t < T$. Let Y be an Ornstein Uhlenbeck process:

$$dY(t) = \alpha(\bar{Y} - Y(t))dt + \omega_Y \left(\beta dB^1(t) + \sqrt{1 - \beta^2} dB^2(t) \right) \tag{1}$$

$$Y(0) = y_0 \in \mathbb{R}.$$

$\beta \in [-1, 1], \alpha > 0, \omega_Y > 0, \text{ and } \bar{Y} \in \mathbb{R}$. We call Y a state process, because it determines an investment opportunity set in our portfolio problem. There is one riskless asset and one risky asset. Suppose the price A_0 of the riskless asset satisfies $dA_0(t) = qA_0(t)dt, A_0(0) = 1$,

Where $q \geq 0$ is constant. The risky asset price A satisfies the stochastic differential equation

$$dA(t) = A(t)\gamma(Y(t))dt + A(t)\omega dB^1(t), A(0) = a > 0, \tag{2}$$

Where $\gamma: \mathbb{R} \rightarrow \mathbb{R}$ satisfies $(\gamma(y) - q) / \omega = y$ for $y \in \mathbb{R}$. Then (2) can be written by

$$dA(t) = A(t)(q + \omega Y(t))dt + A(t)\omega dB^1(t).$$

We consider the division between the riskless asset and the risky assets. Let $\mathbb{N}^2(t_0, t_1)$ be a set of $\mathbb{Z}(t)$ -progressively measurable processes $\sigma: \psi \times [t_0, t_1] \rightarrow \mathbb{R}$ such that

$$P\left(\int_{t_0}^{t_1} \sigma(t)^2 dt < \infty\right) = 1 \tag{3}$$

We call an element of $\mathbb{N}^2(t_0, t_1)$ a portfolio strategy. We regard $\sigma_i(t)$ as a fraction of the wealth invested in the risky asset at time t . The wealth process U^σ corresponding to $\sigma \in \mathbb{N}^2(0, T)$ is given by

$$U^\sigma(0) = u_0 > 0 \text{ and}$$

$$dU(t) = U(t)[\sigma(t)(\gamma(Y(t)) - q) + q]dt + U(t)\sigma(t)\omega dB^1(t)$$

$$= U(t)[\sigma(t)\omega Y(t) + q]dt + U(t)\sigma(t)\omega dB^1(t). \tag{4}$$

There is incompleteness in the sense that there are some random processes that are not replicated by the self-financing portfolio strategy σ . The investor maximizes the expected utility of his wealth at terminal date T . We assume that the investor has a power utility function with a relative risk aversion coefficient δ :

$$\max_{\sigma \in Q_\delta(0, T)} E \left[\frac{U^\sigma(T)^{1-\delta}}{1-\delta} \right]. \tag{5}$$

Here Q_δ denotes the set of admissible portfolio strategies defined as follows. A Stochastic process σ is said to be an admissible portfolio strategy on $[t_0, t_1]$ if

- (a) $\sigma \in \mathbb{N}^2(t_0, t_1)$, when $0 < \delta < 1$
 (b) For some function $\tilde{\sigma} : [0, T] \times \mathbb{R} \rightarrow \mathbb{R}$ satisfying the linear growth condition ,
 $\sigma(t) = \tilde{\sigma}(t, Y(t))$ on $[t_0, t_1]$, when $\delta > 1$.

The set of all admissible strategies on $[t_0, t_1]$ is denoted by $Q_\delta[t_0, t_1]$. The choice of our set of portfolio strategies seems to be restrictive.

Because of incompleteness there is no unique equivalent martingale measure, and we cannot apply the so-called martingale approach directly. It is thus common to apply the dynamic programming approach using Hamilton-Jacobi-Bellman equation. Let

$$K(t, u, y; \sigma) = E^{t, u, y} \left[\frac{U^\sigma(T)^{1-\delta}}{1-\delta} \right],$$

Here and in the sequel, we use the notation $E^{t, u, y}[\cdot] = E[\cdot | U(t) = u, Y(t) = y]$.

Let $S = [0, T] \times (0, \infty) \times \mathbb{R}$. We then define $\zeta : S \rightarrow \mathbb{R}$ by

$$\zeta(t, u, y) = \sup_{\sigma \in Q_\delta(t, T)} K(t, u, y; \sigma).$$

The function ζ is called a value function. The Hamilton-Jacobi-Bellman equation related to the problem (5) is $\sup_{\sigma \in \mathbb{R}} C^\sigma H(t, u, y) = 0$ (6)

With the boundary condition

$$H(T, u, y) = \frac{u^{1-\delta}}{1-\delta}, \tag{7}$$

Where $C^\sigma H(t, u, y) = H_t + u(\sigma\omega y + q)H_u + \alpha(\bar{Y} - y)H_y$

$$+ \frac{1}{2}u^2\sigma^2\omega^2H_{uu} + \frac{1}{2}\omega_y^2H_{yy} + \omega_y u \sigma \omega \beta H_{uy}.$$

It is well-known from Kim and Omberg and others that the function H is separable and has the following form: $H(t, u, y) = \frac{u^{1-\delta}}{1-\delta} g(t, y)$, (8)

Where $g(t, y) = \exp \left\{ p(t) + s(t)y + \frac{1}{2}r(t)y^2 \right\}$

With the boundary conditions $p(T) = s(T) = r(T) = 0$.

It follows from the first order condition for (6) that the candidate optimal portfolio strategy is given by $\sigma^*(t) = \frac{1}{\delta} \frac{Y(t)}{\omega} + \frac{1}{\delta} \frac{\beta\omega_y}{\omega} (s(t) + r(t)Y(t))$. (9)

Substituting this conjectured solution into the Hamilton-Jacobi-Bellman equation, we obtain the differential equation for $p(\cdot), s(\cdot)$, and $r(\cdot)$ as follows:

$$\dot{r}(t) = -\omega_y^2 \left(\frac{1-\delta}{\delta} \beta^2 + 1 \right) r(t)^2 - 2 \left(\frac{1-\delta}{\delta} \omega_y \beta - \alpha \right) r(t) - \frac{1-\delta}{\delta} \tag{10}$$

$$\dot{s}(t) = -\omega_y^2 \left(\frac{1-\delta}{\delta} \beta^2 + 1 \right) s(t)r(t) - \left(\frac{1-\delta}{\delta} \omega_y \beta - \alpha \right) s(t) - \alpha \bar{Y} r(t) \tag{11}$$

$$\dot{p}(t) = -\frac{1}{2} \omega_y^2 \left(\frac{1-\delta}{\delta} \beta^2 + 1 \right) s(t)^2 - \frac{1}{2} \omega_y^2 r(t) - \alpha \bar{Y} s(t) - (1-\delta)q \tag{12}$$

3. Example:

Nine type -2 diabetic patients and nine subjects with normal glucose tolerance participated in the study. Each participant took part in examination in an oral glucose

challenge (75g/30ml) [2], [4] & [5]. The tests were performed in the morning after an overnight fast. After drawing basal blood specimens at 0 minutes, oral glucose was administered. Basal concentrations of immunoreactive GLP-1 were higher in type -2 diabetic patients; whereas GLP- 1 integrated incremental responses after oral glucose were lower than in normal subjects. The peak concentrations reached, however, were similar in type-2 diabetic patients and normal subjects in Figure 2.

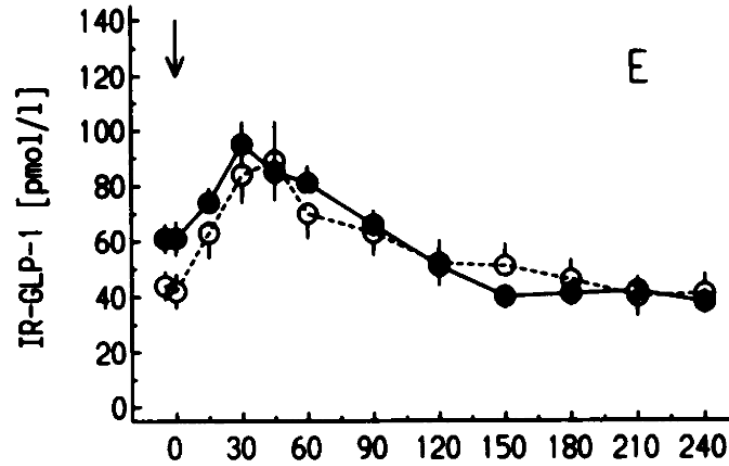


Figure 1: Incretia Activity of Glucagon Like Peptide - 1 With Type - 2 Diabetes Patients

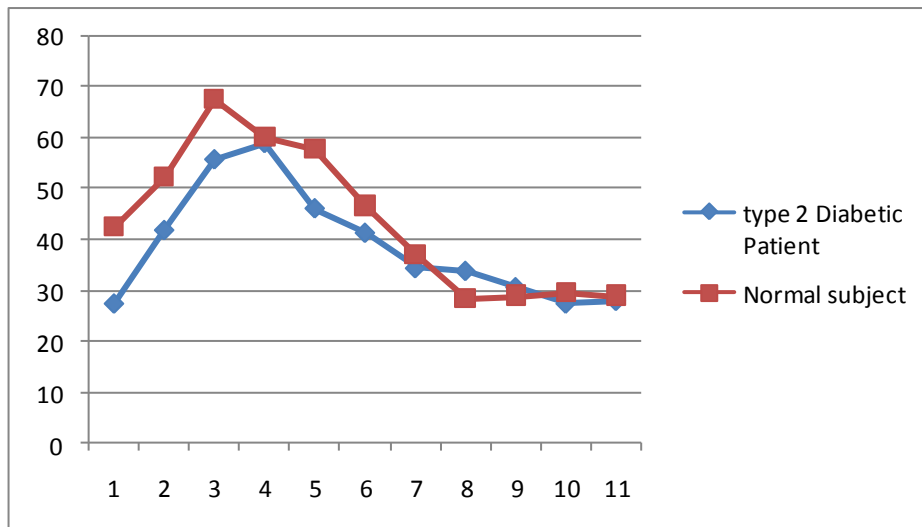


Figure 2: Incretia Activity of Glucagon Like Peptide - 1 With Type - 2 Diabetes Patients (Using Normal Distribution)

4. Conclusion:

The Mathematical Model also stresses the same cumulative effects of type-2 diabetic patients and in normal subjects which are beautifully fitted with Hamilton Jacobi Bellmann equation to the Black Scholes SDE. (Figure 1) The Results of these analyses indicate that in Glucagon like peptide -1 integrated incremental response after oral glucose, the type-2 diabetic patients and normal subjects are similar in the peak concentrations (Figure1 & Figure2). The results exactly related with the mathematical and medical report. In this paper, the problem is investigated by considering the boundary condition of Hamilton Jacobi Bellmann equation. The result coincides with the mathematical and medical report.

5. References:

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