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ON PRIME LABELING OF HERSCHEL GRAPH

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Abstract:

A graph G with vertex set V is said to have a prime labeling if its vertices are labeled with distinct integers $1, 2, 3, \ldots . |V|$ *such that for each xy the labels assigned to x and y are relatively prime. A graph which admits prime labeling is called a prime graph.In this paper, we investigate prime labeling of Herschel graph. We also discuss prime labeling in the context of some graph operations namely Fusion, Duplication, Switching and Path union*

Key Words: Prime Labeling, Fusion, Duplication, Switching & Path Union

1. Introduction:

In this paper, we consider only finite simple undirected graph. The graph G has vertex set $V = V(G)$ and the edge set $E = E(G)$. The set of vertices adjacent to a vertex u of G is denoted by $N(u)$. For notations and terminology we refer to J.A. Bondy and U.S.R. Murthy [1]. In the present work H_s denotes the Herschel graph with 11 vertices and 18 edges.we give brief summary of definitions which are useful for the present investigation. Enough literatures available in printed as well as electronics form on different types of graph labeling and more than 1000 research papers have been published so far in past four decades. A current survey of various graphs labeling problem can be found in [7] (Gallian J, 2009)

Following are the common features of any graph labeling problem.

- ✓ A set of numbers from which vertex labels are assigned.
- ✓ A rule that assigns value to each edge.
- ✓ A condition that these values must satisfy.

The notion of prime labeling was introduced by Roger Entringer and was discussed in a paper by A. Tout (1982 P 365-368) [2]. Many researches have studied prime graph for example in H.C. Fu (1994 P 181-186) [5] have proved that path P_n on n vertices is a prime graph.

T.O Dertsky (1991 P 359-369) [4] have proved that the cycle C_n on n vertices is a prime graph. S.M. Lee (1998 P 59 -67) [3] have proved that wheel W_n is a prime graph iff n is even. Around 1980 Roger Entringer conjectured that all tress have prime labeling, which is not settled till today. The prime labeling for planner grid is investigated by M. Sundaram (2006 P205-209) [6]. In [8] S. K. Vaidhya and K. K. Kanmani) have proved that the prime labeling for some cycle related graphs. In [9] S. Meena and K. Vaithilingam, Prime Labeling for some Helm related graphs.We will provide brief summary of definitions and other information which are necessary for the present investigations.

Definition1: If the vertices of the graph are assigned values subject to certain conditions then it is known as (vertex) graph labeling.

Definition2: Let $G = (V(G), E(G))$ be a graph with n vertices. A bijection $f: V(G) \to \{1, 2, \ldots n\}$ is called a Prime labeling if for each edge $e = uv$, $gcd(f(u), f(v)) = 1$. A graph which admits prime labeling is called a prime graph.

Definition 3: An independent set of vertices in a graph G is a set of mutually non-adjacent vertices.

Definition 4: Let u and $v$be two distinct vertices of a graph G. A new graph G_1 is constructed by fusing (identifying) two vertices u and v by a single vertex x in G_1 such that every edge which was incident with either u (or) v in G now incident with $x$in G_1.

Definition5: Duplication of a vertex v_k of a graph G produces a new graph G_1 by adding a vertex $v_k^{'}$ with $N(v_k^{'}) = N(v_k)$. In other words, a vertex $v_k^{'}$ is said to be a duplication of v_k if all the vertices which are adjacent to v_k are now adjacent to $v_k^{'}$.

Definition6: A vertex switching G_v of a graph G is obtained by taking a vertex $v$of G, removing the entire edges incident with v and adding edges joining v to every vertex which are not adjacent to v in G.

Definition7: Let $G_{1,}G_2, G_3, \dots G_n,\ n \geq 2$ be n copies of a fixed graph G. The graph obtained by adding an edge between G_i and G_{i+1} for $i = 1, 2, \dots n - 1$ is called the path union of$G$.

Definition 8: The Herschel graph H_s is a bipartite undirected graph with 11 vertices and 18 edges.

Example:

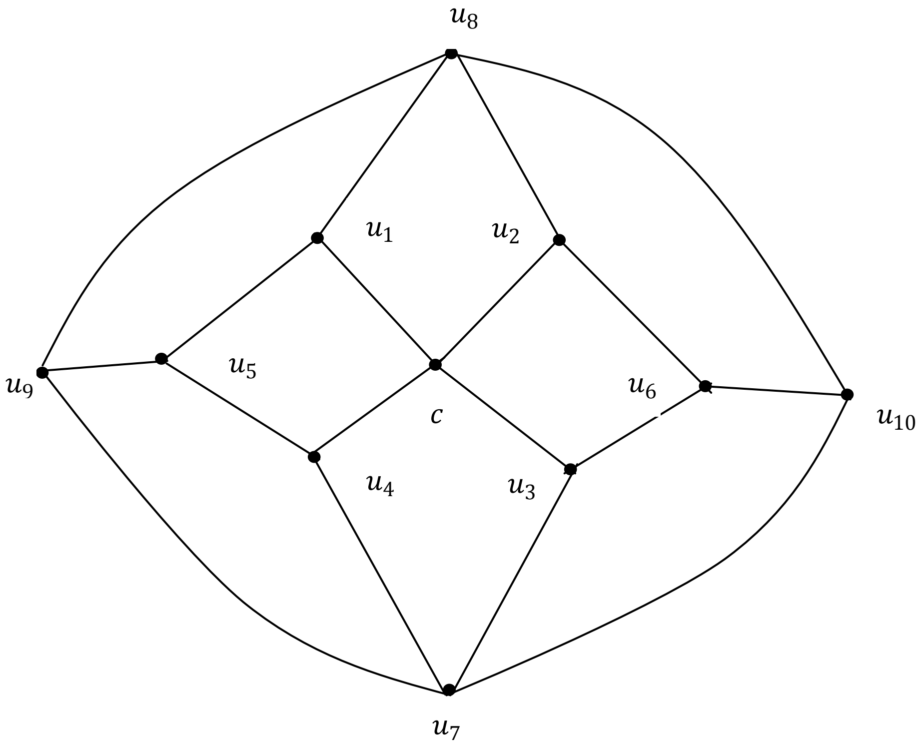


Figure 1: A Herschel Graph

Proposition 1:

The Herschel graph H_s is a prime graph.

Proof:

Let H_s be the Herschel graph with 11 vertices and 18 edges and let$c$ be the centre of the Herschel graph. Then $|V(H_s)| = 11$and $|E(H_s)| = 18$

Let $f(c) = 1$and$f(u_i) = 2i$ for $1 \leq i \leq 4$where$u_i$sare adjacent to c.

(i.e)

$$f(u_1) = 2$$
$$f(u_2) = 4$$
$$f(u_3) = 6$$
$$f(u_4) = 8$$

Next, we label the vertex u_5 which is adjacent to u_1 and$u_4$having even label.

$$\therefore \text{Let} f(u_5) = 3$$

We label the vertexu_6 which is adjacent to u_2 and u_3 having even label

$$\therefore \text{Let} f(u_6) = 5$$

Similarly, u_7 is adjacent to u_3 and u_4 both having even label

$\therefore f(u_7) = 7$ and u_8 is adjacent to u_1 and u_2 and having even label

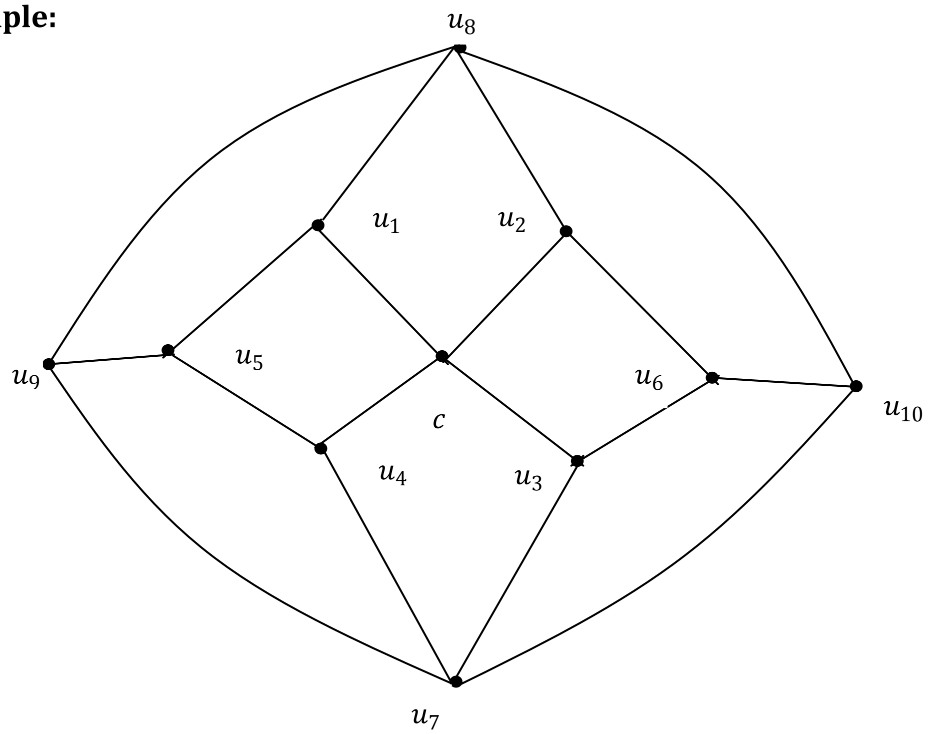
$$\therefore f(u_8) = 9$$

Finally, Let$f(u_9) = 10$and$f(u_{10}) = 11$

Hence, for each$e = cu_i \in H_s$, $\gcd\big(f(c), f(u_i)\big) = 1$ and for the edge $e = u_iu_j \in H_s$, $\gcd\Big(f(u_i), f(u_j)\Big) = 1$

Hence H_s admits a prime labeling.

Example:



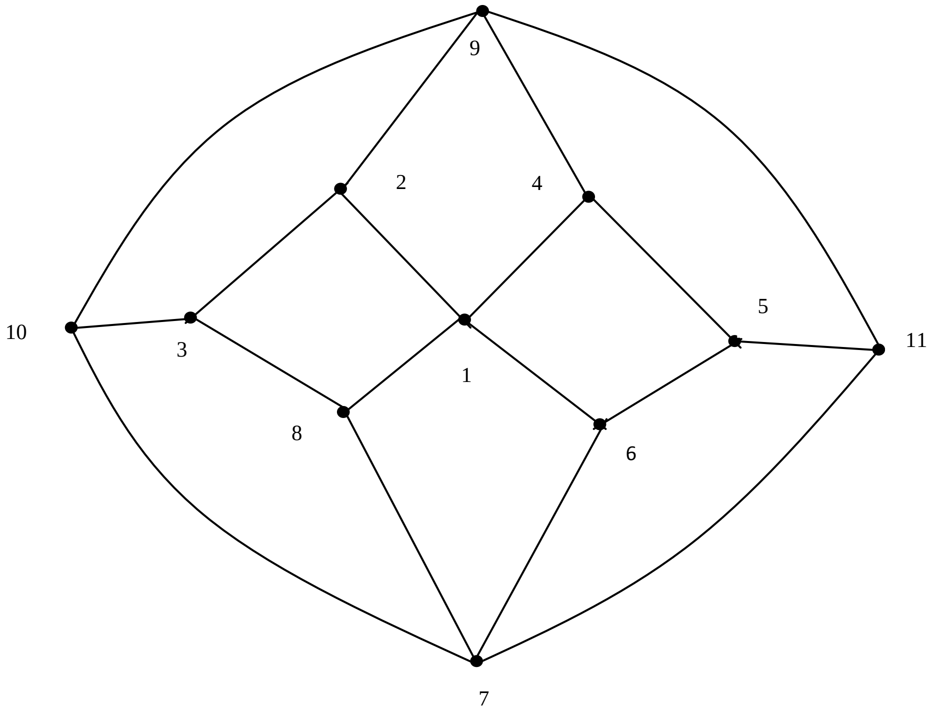


Figure 2: The Herschel graph is a prime graph

Proposition 2:

The Fusion of two adjacent vertices of degree 3 in a Herschel graph is a prime graph.

Proof:

Let H_s be the Herschel graph with $|V(H_s)| = 11$ and $|E(H_s)| = 18$

Let $c$be the centre of the Herschel graph and it has 3 vertices of degree 4 and 8 vertices of degree 3.

Let G be the graph obtained by fusing any two adjacent vertices of degree 3 in the Herschel graph. Then $|V(G)| = 10$

Define a label $f: V(G) \to \{1, 2,3, \dots \dots 10\}$

Such that $f(c) = 1$

and$f(u_i) = 2i$ for $1 \le i \le 4$ where$u_i s$ are adjacent to the centre c.

(i.e.)$f(u_1) = 2$

$$f(u_2) = 4$$
$$f(u_3) = 6$$
$$f(u_4) = 8$$

Next, we label the vertexu_5 which is adjacentu_1 and u_4 both of them having even label and the vertex $u_6$which is adjacent to u_2 and u_3 of even label.

Therefore, Let$f(u_5) = 3$,

$$f(u_6) = 5$$

Similarly, $f(u_7) = 7$,

$$f(u_8) = 9$$

Finally, we assign the label the vertex u_9 with the remaining label 10,

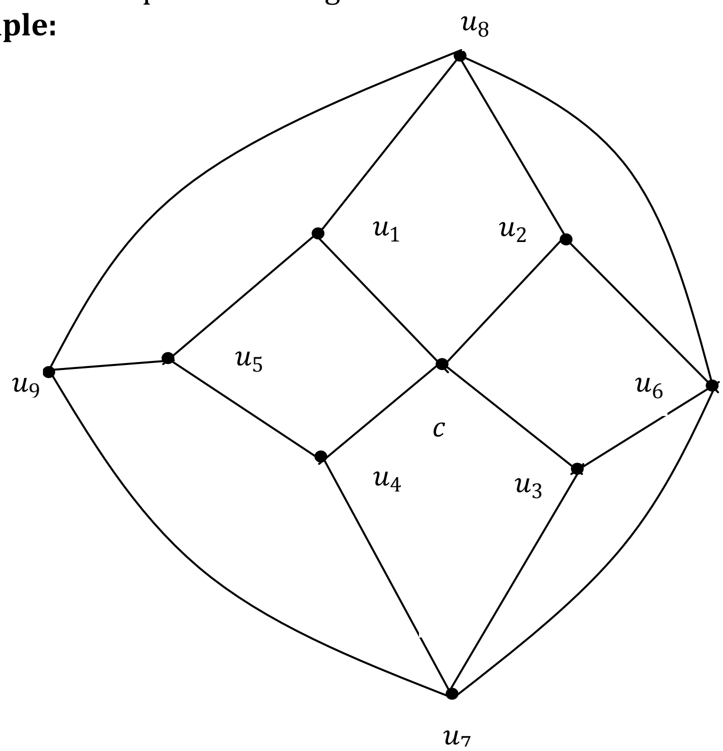
(i.e.)$f(u_9) = 10$

Now, for each edge $e = u_i u_j \in G gcd\left(f(u_i), f(u_j)\right) = 1$

and for the edges $e = cu_i \in G$, $gcd(f(c), f(u_i)) = 1$

Hence G admits prime labeling.

Example:



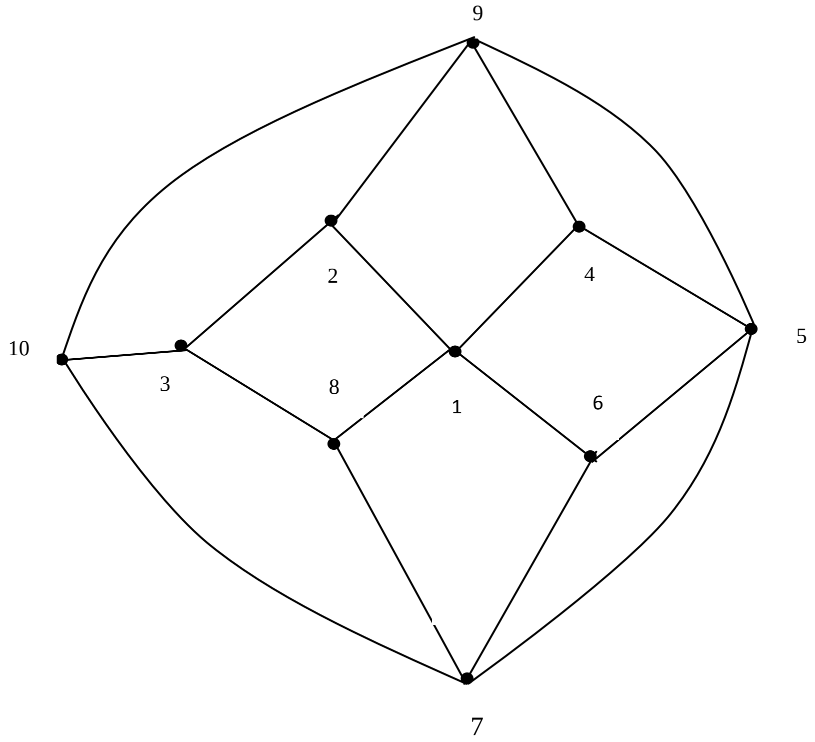


Figure 3: Fusion of the u_6 and u_{10} in a Herschel graph is a prime graph.

Proposition 3:

The Duplication of any vertex of degree 3 in a Herschel graph is a prime graph.

Proof:

Let H_s be the Hershel graph with $|V(H_s)| = 11$ and $|E(H_s)| = 18$

Let c be the centre vertex and u_k be any vertex of degree 3, $u_k^{'}$ be the duplication of the vertex u_k in the Herschel graph H_s.

Let G_k be the graph obtained by after duplicating the vertex u_k of degree 3 in H_s. Then $|V(G_k)| = 12$ Define a label $f: V(G_k) \to \{1, 2, \ldots ..12\}$ such that $f(c) = 1$

$f(u_k^{'}) = 12$ where $u_k^{'}$ is the duplicating vertex of u_k.

and $f(u_i) = 2i$ for $1 \leq i \leq 4$, where $u_i s$ are the adjacent to c.

(i.e.) $f(u_1) = 2$

$f(u_2) = 4$

Let $f(u_3) = 6$

$f(u_4) = 8$

Let $f(u_5) = 3$ since u_5 is adjacent to u_1 and u_4 having even label

$f(u_6) = 5$ since u_6 is adjacent to u_2 and u_3 having even label

$f(u_7) = 7$ since u_7 is adjacent to u_3, u_4, u_9, u_{10} and u_k

$f(u_8) = 11$ since u_8 is adjacent to u_1, u_2, u_9, u_{10} and u_k

$f(u_9) = 10$ since u_9 is adjacent to u_5, u_7, u_8 having even label

$f(u_k) = 9$ Here we duplicating u_k in H_s as $u_k^{'}$.

Now, for each edge $e = cu_i \in G_k$, $\gcd(f(c), f(u_i)) = 1$

and for the edge $e = u_i u_j \in G_k$, $\gcd\left(f(u_i), f(u_j)\right) = 1$

Hence G_k admits a prime labeling.

Example:

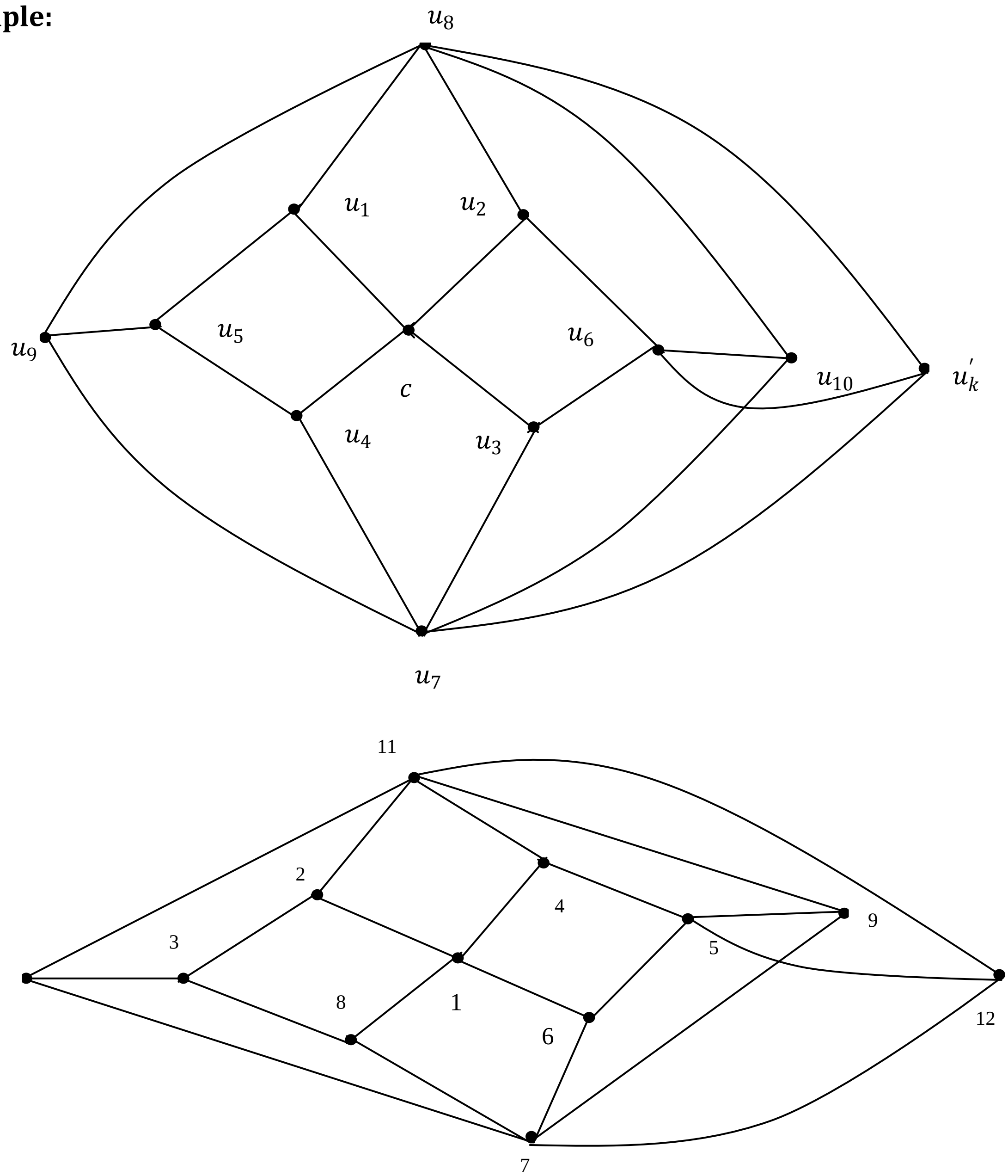


Figure: 4 Duplication of the vertex u_{10} of degree 3 in H_s is a prime graph

Proposition 4:

Switching the centre vertex c in the Herschel graph H_s is a prime graph.

Proof:

Let H_s be the Herschell graph with $V|H_s| = 11$ and $|E(H_s)| = 18$

Let the centre vertex $c$be the switching vertex and $G_c$be the new graph obtained by switching the centre vertex c.

Clearly $|V(G_c)| = 11$and$|E(G_c)| = 19$

Define a label $f: V(G_c) \to \{1, 2, \dots \dots 11\}$

such that $f(c) = 1$and$f(u_i) = 2i$ for $1 \leq i \leq 4$ where $u_i s$ are adjacent to c.

(i.e)$f(u_1) = 2$

$f(u_2) = 4$

$f(u_3) = 6$

$f(u_4) = 8$

Let $f(u_5) = 3$Sinceu_5 is adjacent to u_1 and u_4 of even labels

$f(u_6) = 5$Since u_6 is adjacent to u_2 and u_3 of even labels

$f(u_7) = 7$Since u_7 is adjacent to u_3 and u_4 of even labels

$f(u_8) = 9$Since u_8 is adjacent to u_1 and u_2 even labels

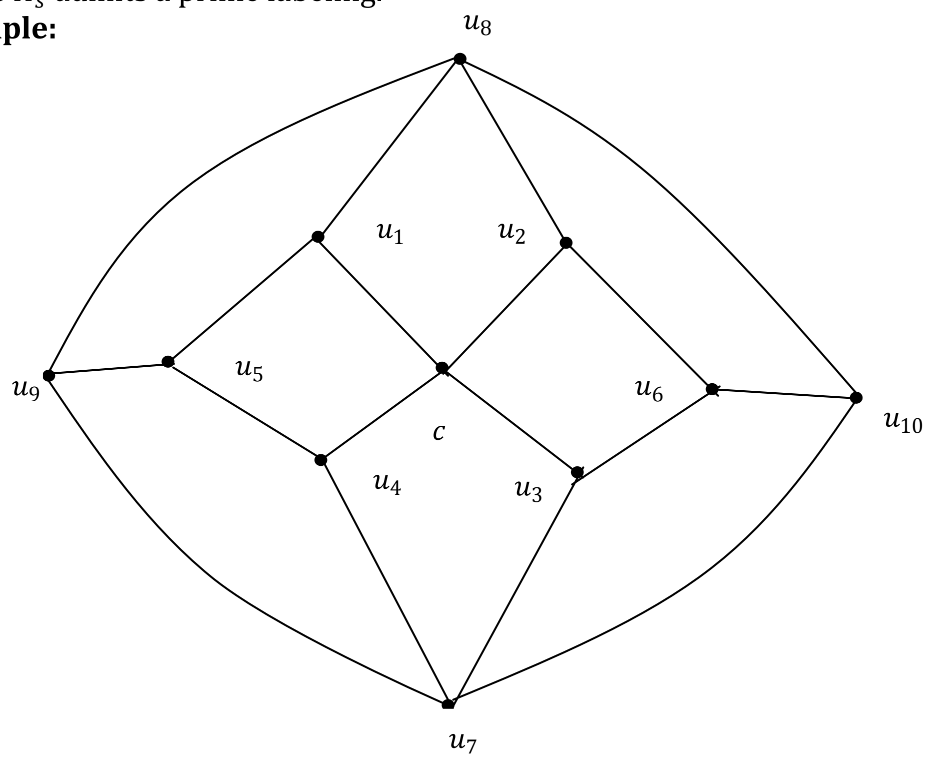
$f(u_9) = 10$Sinceu_9 is a adjacent to u_7 and u_8 even labels

Finally,$f(u_{10}) = 11$

For each edge $e = u_i u_j \in G_c$, $gcd\left(f(u_i), f(u_j)\right) = 1$

Hence H_s admits a prime labeling.

Example:



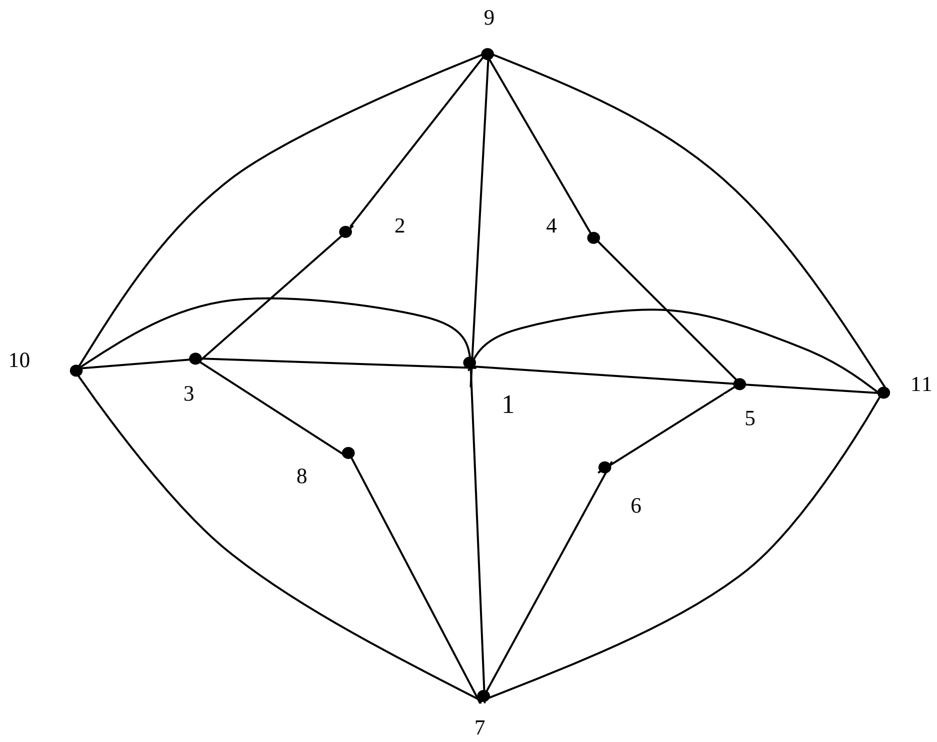


Figure 5: Switching of the center vertex c in H_s is a prime graph.

Proposition 5:

The graph obtained by path union of two pieces of Herschel graph H_s is a prime graph.

Proof:

Consider, two copies of Herschel graph H_s and H_s^* respectively
Then$|V(H_s)| = 11$ and $|E(H_s)| = 18$
and$|V(H_s^*)| = 11$ and $|E(H_s^*)| = 18$
Let G_k be the graph obtained by the path union of two pieces of Herschel graphs H_s andH_s^* . $V(G_k) = V(H_s) \cup V(H_s^*)$ $\therefore |V(G_k)| = 22$
$E(G_k) = E(H_s) \cup E(H_s^*) \cup \{u_k v_k\} \therefore |E(G_k)| = 37$
we assign the labels $1, 2, 3, \ldots \ldots 11$ for H_s and$13, 14, \ldots \ldots .23forH_s^*$
so that $\therefore |V(G_k)| = 22$
Define a label $f: V(G_k) \to \{1, 2, 3, \ldots . . 11, 13, 14, \ldots \ldots 23\}$

LabelingH_s
Let $f(c_1) = 1$
$f(u_i) = 2i$for$1 \leq i \leq 4$
(i.e.) $f(u_1) = 2$
$f(u_2) = 4$
$f(u_3) = 6$
$f(u_4) = 8$
Let$f(u_5) = 5$
$f(u_6) = 7$
$f(u_7) = 9$
$f(u_8) = 10$
$f(u_{10}) = 11$

LabelingH_s^*
Let$f(c_2) = 13$
$f(v_1) = 20$
$f(v_2) = 16$
$f(v_3) = 14$
$f(v_4) = 18$
Let $f(v_5) = 15$
$f(v_6) = 23$
$f(v_7) = 17$
$f(v_8) = 19$
$f(v_9) = 22$
$f(v_{10}) = 21$

since, for each edge $e = c_1 u_i \in G_k$, gcd$(f(c_1), f(u_i)) = 1$
andfor the edge$e = u_i u_j \in G_k$, gcd$(f(u_i), f(u_j)) = 1$
also for the edge $e = c_2 v_i \in G_k$, gcd$(f(c_2), f(v_i)) = 1$
and for the edge$e = v_i v_j \in G_k$,gcd$(f(v_i), f(v_j)) = 1$
Hence $G_k$admits prime labeling.

Example:

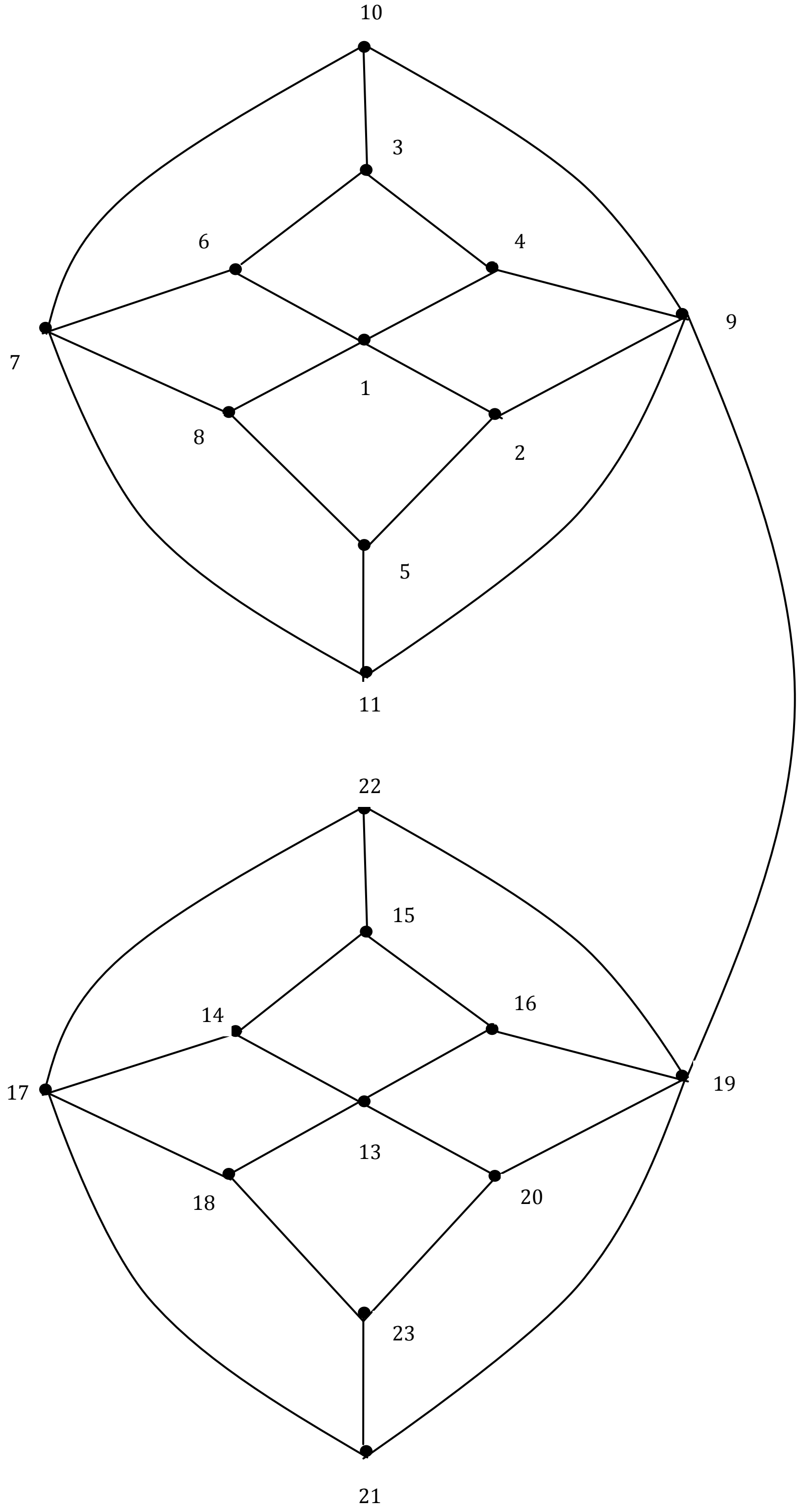


Figure 6: Path unions of two pieces of Herschel graph is a prime graph

Conclusion:

Here, wehave investigate a five results corresponding to prime labeling on some special graph, namely Herschel graph. Analogous work can be carried out for other families and in the context of different types of graph labeling techniques.

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