International Journal of Engineering Research and Modern Education (IJERME)
Impact Factor: 6.525, ISSN (Online): 2455 - 4200
(www.rdmodernresearch.com) Volume 2, Issue 1, 2017

RAINDROPS FROM MOTIONLESS CLOUD USING PROPORTIONALITY AND GEOMETRIC SIMILARITY IN MATHEMATICAL MODELING

Dr. S. Jayakumar* & S. Geetha**

* Head, Department of Mathematics, Bon Secours College for Women, Villar Bypass, Thanjavur, Tamilnadu

** Assistant Professor, Department of Mathematics, Bon Secours College for Women, Villar Bypass, Thanjavur, Tamilnadu

Cite This Article: Dr. S. Jayakumar & S. Geetha, "Raindrops from Motionless Cloud Using Proportionality and Geometric Similarity in Mathematical Modeling", International Journal of Engineering Research and Modern Education, Volume 2, Issue 1, Page Number 53-57, 2017.

Abstract:

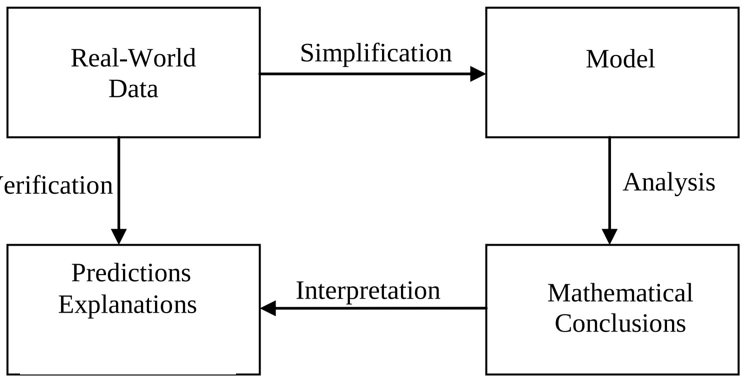
Mathematical Model is an idealization of the real world Phenomenon and never a completely accurate representation. Any Model has its limitations a good one can provide valuable results and conclusions. Mathematical Model as a mathematical construct designed to study a particular real world systems or behavior of Interest. The Model allows us to reach mathematical conclusions about the behavior; These conclusions can be interpreted to help a decision maker plan for the future. Most models simplify reality. Generally, models can only approximate real-world behavior. One powerful simplifying relationship is proportionality. The graphical models representing population size, drug concentration in the bloodstream, various financial investment, and the distribution of ears between two cities for a rental company. A Mathematical model as a mathematical Construct designed to study a particular real-world system or phenomenon. It includes graphical, symbolic, simulation, and experimental constructs. The Main objective of this paper is to the construction of mathematical models in vehicular stopping Distance and Raindrops from a motionless cloud using Geometric similarity, Proportionality. The ultimate goal is to test the rule, and suggest another rule if it fails. Geometric similarity is a concept related to proportionality and can be useful to simplify the Mathematical modeling. Likewise, to find the terminal velocity of a raindrop from a motionless cloud the only forces acting on the raindrop are gravity and drag. The principal of geometric similarity suggests a conveicut method for testing to determine whether it holds among of collection of objects. There are existing mathematical models that can be identified with some particular real-world phenomenon by using the mathematical conclusions. The Mathematics involved may be so complex and intractable that there is little hope of analyzing or so loving the model.

Key Words: Mathematical Model, Simplification, Proportionality Geometric Similarity, Interpretation, Variables and Sub Models, Vehicular Stopping Distance, Raindrop, Force of Gravity & Air Resistance.

Introduction:

Mathematical model is an idealization of the real-world phenomenon and never a completely accurate representation. Although any model has its limitations, a good one can provide valuable results and conclusion. Mathematical model as a mathematical construct designed to study a particular real-world system or behavior of interest. The model allows us to re ach mathematical conclusions about the behavior, as illustrated in Figure 1.1.These conclusions can be interpreted to help a decision maker plan for the future.

Figure 1.1: A flow of the modeling process beginning with an examination of real –world data



Proportionality:

Most models simplify reality. Generally, models can only approximate real-world behavior. One very powerful simplifying relationship is proportionality. Two variables y and x are proportional (to each other) if one is always a constant multiple of the other- that is, if

$$y=kx \qquad (1.1)$$

for some nonzero constant k. We write $y \alpha x$.

Modeling Change with Difference Equations:

For a sequence of numbers $A=\{a_0,a_1,a_2,a_3,\ldots\}$ the first difference are

$$\Delta a_0=a_1-a_0$$
$$\Delta a_1=a_2-a_1 \qquad (1.2)$$
$$\Delta a_2=a_3-a_2$$
$$\Delta a_3=a_4-a_3$$

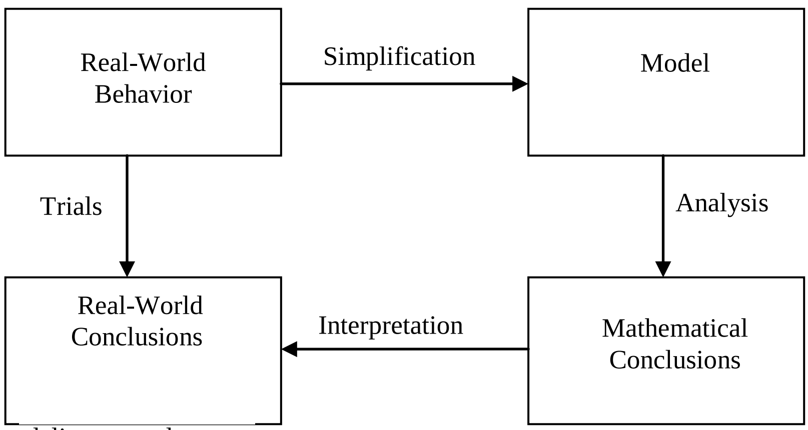
For each positive integer *n*, the *n*th first difference is $\Delta a=a_n+1-a_n$

Graphical models representing population size, drug concentration in the bloodstream, various financial investments, and the distribution of cars between two cities for a rental company. Examine more closely the process of mathematical modeling. To gain an understanding of the process involved in mathematical modeling, consider the two worlds depicted in Figure 2.1. Suppose to understand some behavior or phenomenon in the real world.

Figure 1.2: The Real and Mathematical Worlds

Real World Systems	Mathematical World
Observed behavior or phenomenon	Models Mathematical operations and rules Mathematical conclusions

Figure 1.3: Reaching conclusions about the behavior of real-world system



The following are the modeling procedure:

- ✓ Through observation, identify the primary factors involved in the real-world behavior, possibility making simplifications.
- ✓ Conjecture tentative relationships among the factors.
- ✓ Apply mathematical analysis to the resultant model.
- ✓ Interpret mathematical conclusions in terms of the real-world problem.

Figure 1.4: The Modelling Process as a Closed System

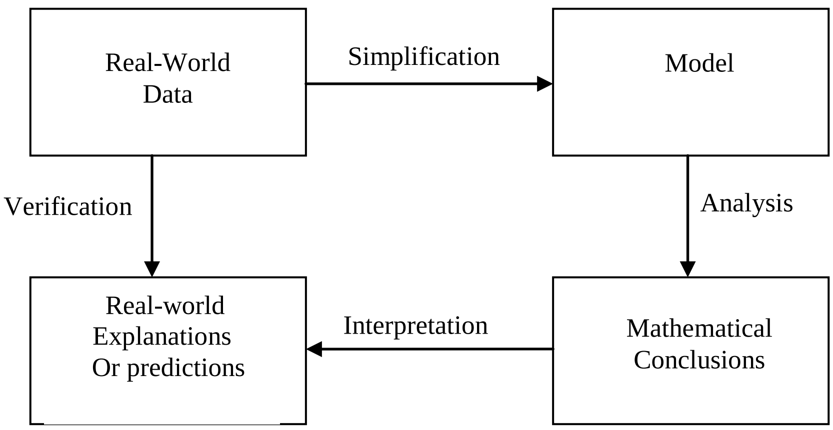
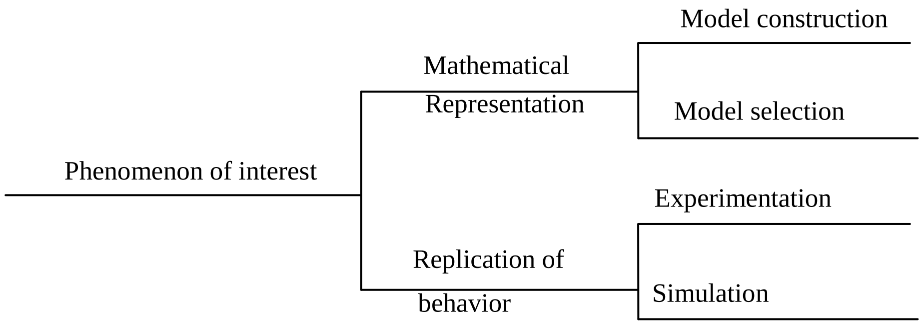


Figure 1.5: The Nature of the Model



So large (in terms of the number of factors involved) that it is impossible to capture all the information in a single mathematical model.

Construction of a Mathematical Model:

Step 1: Identify the problem

Step 2: Make assumptions

a. Identify and classify the variables.
b. Determine interrelationships between the variables and sub models.

Step 3: Solve the model.

Step 4: Verify the models

a. Does it address the problem?
b. Test it with real-world data.

Step 5: Implement the model.

Step 6: Maintain the model.

Modelling Vehicular Stopping Distance:

Allow one car length for every 10 miles of speed under normal driving conditions, but more dis adverse weather or road conditions. One way to accomplish this is to use the 2-second rule for m the correct following distance no matter what your speed. To obtain that distance, watch the vehicular of you pass some definite point on the highway, like a tar strip or overpass shadow. Then count to "One thousand and one, one thousand and two;" that is 2 seconds. If you reach the mark before you saying those words, then you are following too close behind. The preceding rule is implemented easily enough, but how good is it?

Problem Identification:

Our ultimate goal is to test this rule and suggest another rule if however, the statement of problem-How good is the rule? -- is vague. Consider the following problem statement: a vehicles total stopping distance as function of its speed.

Assumption:

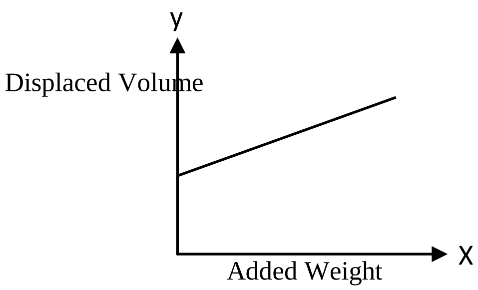
Being the analysis with a rather obvious model for total stopping distance

Total Stopping Distance = Reaction Distance + Braking Distance

By reaction distance, we mean the distance the vehicle travels from the instant the driver per need to stop to the instant when the brakes are actually applied. Braking distance is the distance required for the brakes to bring the vehicle to a complete stop. First let's develop a sub model for reaction distance. The reaction distance is a function of variables, and we start by listing just two of them:

$$\text{Reaction Distance} = f\,(\text{Response Time, Speed}) \qquad (1.3)$$

A straight –line relationship exists between displaced volume and total weight, but it is not proportionality because the line fails to pass through the origin.



A proportionality relationship may, however, be a reasonable simplifying assumption, depending on the size of the y-intercept and the slope of the line. The domain of the independent variable can also be significant since the relative error.

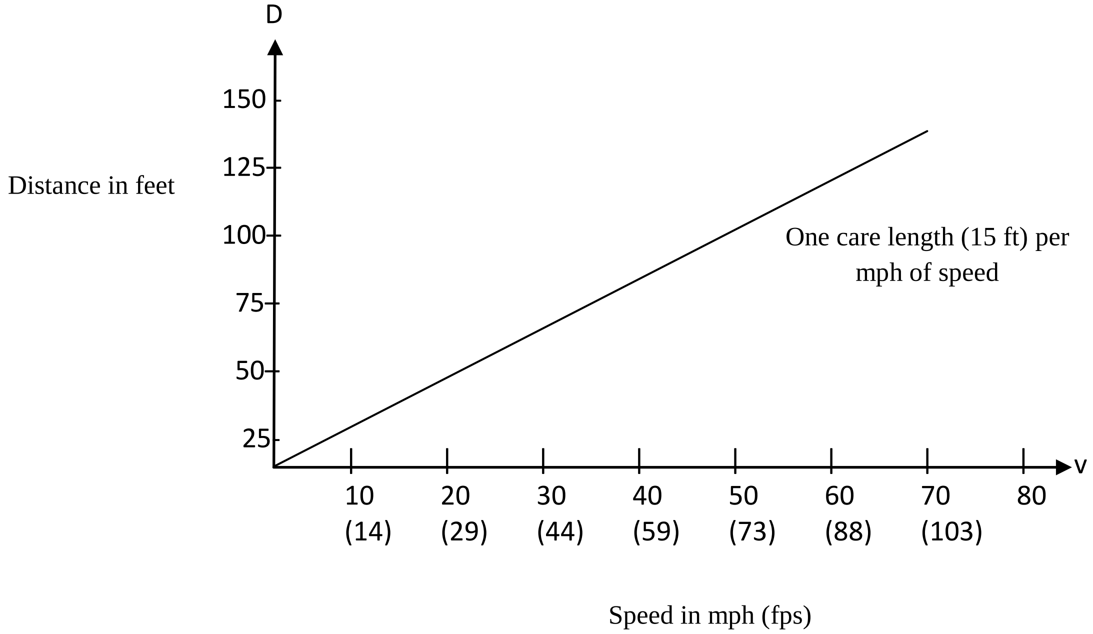
is greater for small values of *x*.

$$\frac{y_a - y_p}{y_a} \qquad (1.4)$$

The general rule that allows car length for every mph of speed. It was also stated that this rule is the same as allowing for seconds between cars. The rule in fact difference from one another (at least for most cars). For rules to be same, at mph both should allow one car length. 1 car length = distance = (speed in ft/ sec) (2sec)= (10 miles/hr) (8080 ft/mi) (1 h r/ 3600 sec) (2 sec)= 29.33 ft). Lets interpret the one-car-length rule geometrically. If we assume a car length of 15 ft and plot a rule, we obtain the graph shown in Figure 2.13, which shows that the distance allowed by the is proportional to the speed. In fact, if we plot the speed in feet per second, the constant of proportionality has the units seconds and represents the total time for the equation D=kv to make. More ever, in the case of a 15ft care, we obtain a constant of proportionality as follows:

K=15 ft/10mph = 1 5ft/52,800 ft /3600 sec =960/88 sec) (1.6)

Figure 1.7: Geometric Interpretation of the One-Car-Length Rule



Modeling Using Geometric Similarity:

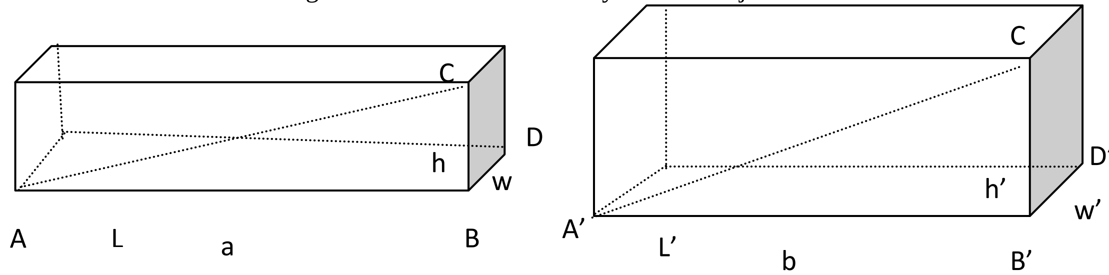
Geometric similarity is a concept related to proportionality and can be useful to simplify the mathematical modeling process.

Geometric Similarity: Two objects are said to be geometrically similar if there is a one-to-one correspondence between points of the objects such that the ratio of distances between corresponding points is constant for all possible Paris of points.

Example: Consider the two boxes depicted in figure 2.1 let L denote the distance between the points A and B in Figure 2.1. and Let L be the distance between the corresponding points A and B in Figure 2.1. Other Corresponding points in the two figures, and the associated distance between the points, are marked the same way. For the boxes to be geometrically similar, it must be true that

L/l = w/w=h/h =k for some constant k>0 (1.7)

Figure 2.1: Two Geometrically Similar Objects



Interpret the result geometrically. In Figure 2.1 consider the triangles ABC and A B C. If the two boxes are geometrically similar, these two triangles must be similar. The same argument can be applied to any corresponding pair of triangles, such as CBD and C B D. Thus corresponding angles are equal for objects that are geometrically similar. In other words, the shape is the same for two geometrically similar objects, and one object is simply an enlarged copy of the other. We can think of geometrically similar objects, and one object is simply an enlarged copy of the other. We can think of geometrically similar objects as scaled replicas of one another, as in an architectural drawing in which all the dimensions are simply scaled by some constant factor K.

One advantage that results when two objects are geometrically similar is a simplification in certain computations, such as volume and surface area. For the boxes depicted in Figure 2.18, consider the following argument for the ration of the volume V an V.

$$V/V = Lwh/L'w'h' = k^3 \quad (1.8)$$

Similarly, the ration of their total surface areas S and S' is given by

$$S/S' = 2Lh+2wh+2wL/2L'h'+2w'h'+2w'L' = k^2 \quad (1.9)$$

Not only are these rations immediately known once the scaling factor k has been specified, but also the surface area and volume may be expressed as a proportionality in terms of some selected characteristic dimension. Let's select the length L as the characteristic dimension. Then with L/L'=k we have.

$$S/s' = k^2 = L^2/L'^2$$

Therefore, $S/L^2 = s'/L'^2 = \text{constant}$

Holds for any two geometrically similar objects. That is, surface area is always proportional to the square of the characteristic dimension length.

$$S \alpha L^2$$

Likewise, volume is proportional to the length cubed.

$$V \alpha L^3$$

Thus, if we are interested in some function depending on an object's length, surface area, and volume, for example, $y=f(L,S,V)$

We could express all the function arguments in terms of some selected characterisitc dimension, such as length, giving, $y=g(L, L^2, L^3)$

Geometric similarity is a powerful simplifying assumption.

Or $Fg = Fd$

Assuming that $Fd \alpha Sv^2$ and that Fg proportional to weight w. Since $m \alpha w$, we have $Fg \alpha m$. Next assume all the raindrops are geometrically similar. This assumption allows us to relate area and volume so that

$$S \alpha L^2 \text{ and } V \alpha L^3$$

For any characteristic dimension L. Thus L $S^{1/2} \alpha V^{1/3}$, which implies

Because weight and mass are proportional to volume, the transitive rule proportionality gives

$$S \alpha m^{2/3}$$

From the equation Fg = Fd, now, $m \alpha m^{2/3}v^2$ Solving for the terminal velocity, we have

$$\Rightarrow \quad M^{1/3} \alpha v^2 t \text{ or } M^{1/6} \alpha vt$$

Therefore, the terminal velocity of the raindrop is proportional to its mass raised to the one-sixth power.

Testing Geometric Similarity:

The Principle of geometric similarity suggests a convenient method for testing to determine whether 2 it holds amount a collection of objects. Because the ratio of distance between corresponding pairs of points be the same for all pairs of points. If the objects in a given collection are geometrically similar. For example, the circles are geometrically similar (because all circles have the same shape, possibly varying only in size), If c denotes the circumference of a circle, d its diameter, and s the length of arc along the circle subtended by a given (fixed) angle 0, then know from geometry that

$$c = \pi d \text{ and } s = (d/2)\theta$$

Thus, for any two circles, $c_1/c_2 = \pi d_1/\pi d_2 = d_1/d_2$

and $S_1/S_2 = (d_1/2)\theta/(d_2/2)\theta = d_1/d_2$

That is, the ratio of distance between corresponding points (points that have the same fixed angle) any two circles is always the ratio of their diameter. This observation supports the reasonableness of the geometric similarity argument for the circles.

References:

1. Kapur .J. N., Mathematical Modeling, Wiley Eastern Limited, New Delhi, 1988.
2. Giordano. F. R., Fox W. P. and Horton. S. B., Weir. M. D, Mathematical Modeling Principles and Applications.
3. Frend. J. E, Mathematical Statistics, Prentice Hall of India.
4. Kapoor. J.N, Mathematical Models in Biology and Medicine, Affiliated East – West Press Pvt., Limited, New Delhi, 1981.
5. Alder. M. D. An Introduction to Mathematical Modeling 2001 Edition
6. Rao. S. S., Optimization Theory and Applications, Wiley Eastern Limited, New Delhi.